Monkeys and Coconuts - Numberphile
https://www.youtube.com/watch?v=U9qU20VmvaU
That example had five sailors and a monkey -- let's go with three sailors here

https://en.wikipedia.org/wiki/The_monkey_and_the_coconuts

In[1]:=
```
Div1 = C0 == 1 + 3 S1
```
Out[1]= C0 == 1 + 3 S1

In[2]:=
```
Div2 = 2 S1 == 1 + 3 S2
```
Out[2]= 2 S1 == 1 + 3 S2

In[3]:=
```
Div3 = 2 S2 == 1 + 3 S3
```
Out[3]= 2 S2 == 1 + 3 S3

In[4]:=
```
Div4 = 2 S3 == 3 S4
```
Out[4]= 2 S3 == 3 S4

In[5]:=
```
Sol2 = Solve[Div2, S1]
```
Out[5]= $\left\{\left\{S1 \to \frac{1}{2}(1 + 3\ S2)\right\}\right\}$

In[6]:=
```
Div1B = Simplify[Div1 /. Sol2][[1]]
```
Out[6]= 2 C0 == 5 + 9 S2

In[7]:=
```
Sol3 = Solve[Div3, S2]
```
Out[7]= $\left\{\left\{S2 \to \frac{1}{2}(1 + 3\ S3)\right\}\right\}$

In[8]:=
```
Div1C = Simplify[Div1B /. Sol3][[1]]
```
Out[8]= 4 C0 == 19 + 27 S3

In[9]:=
```
Sol4 = Solve[Div4, S3]
```
Out[9]= $\left\{\left\{S3 \to \frac{3\ S4}{2}\right\}\right\}$

In[10]:=
```
Div1D = Simplify[Div1C /. Sol4][[1]]
```
Out[10]=

8 C0 == 38 + 81 S4

Linear Diophantine Equation
Wikipedia: In mathematics, a Diophantine equation is an equation, typically a polynomial equation in two or more unknowns with integer coefficients, for which only integer solutions are of interest.

A Nice Diophantine Equation in Number Theory | You Should Learn This Theorem | Math Olympiad
https://www.youtube.com/watch?v=NClzCdCmFiA

If the greatest-common-divisor of the coefficients divides the constant, then there are an infinite number of solutions. (Otherwise, there are none.)
8 C0==38+81 S4
8 (C0+81N)==38+81 (S4+8N)
8 C0 + 8*81*N ==38+81 S4 + 81*8*N

In[11]:=
```
GCD[8, 81]
```
Out[11]=
```
1
```

First step in Euclid Algorithm
81 = 10*8 + 1
Turn around around
1 = 81*1 - 8*10
Bring in (rest of) constant
38 = 81*(1*38) - 8*(10*38)
Get into form of desired equation by changing signs
38 = -81*(-38) + 8*(-380)
C0=-380 and S4=-38 is a solution
Add enough 81's to C0 and the corresponding of 8's to S4 to get the lowest positive integer solution
-380 + 5*81, -38+5*8 --> 25, 2

In[12]:=
```
FindInstance[Div1D, {C0, S4}, Integers]
```
Out[12]=
```
{{C0 → 25, S4 → 2}}
```

In[13]:=
```
FindInstance[Div1D, {C0, S4}, Integers, 3]
```
Out[13]=
```
{{C0 → -10 829, S4 → -1070}, {C0 → -3053, S4 → -302}, {C0 → 10 069, S4 → 994}}
```

In[14]:=
```
FindInstance[Div1D && C0 > 0, {C0, S4}, Integers, 3]
```
Out[14]=
```
{{C0 → 25, S4 → 2}, {C0 → 1402, S4 → 138}, {C0 → 22 300, S4 → 2202}}
```

In[17]:=
```
FindInstance[Div1D && 0 < C0 < 200, {C0, S4}, Integers, 3]
```
Out[17]=
```
{{C0 → 25, S4 → 2}, {C0 → 106, S4 → 10}, {C0 → 187, S4 → 18}}
```