

```
In[49]:= die = RandomInteger[{1, 6}]
```

```
Out[49]=  
2
```

```
In[50]:= SIZE = 1000
```

```
Out[50]=  
1000
```

```
In[67]:= SIDE = 12
```

```
Out[67]=  
12
```

```
In[68]:= dice = RandomInteger[{1, SIDE}, SIZE]
```

```
Out[68]=
```

```
{9, 7, 5, 5, 9, 4, 4, 7, 12, 1, 2, 5, 10, 1, 3, 2, 11, 2, 12, 8, 1, 10, 11, 7, 2, 8, 11, 9, 9, 8,
  5, 9, 12, 10, 2, 11, 12, 3, 9, 9, 6, 11, 9, 3, 2, 8, 8, 12, 8, 9, 12, 11, 1, 12, 4, 2,
  1, 7, 4, 2, 5, 4, 6, 2, 7, 8, 5, 8, 9, 1, 9, 7, 2, 12, 12, 10, 9, 1, 5, 8, 12, 5, 1, 8,
  7, 9, 3, 8, 7, 9, 9, 2, 10, 8, 5, 1, 8, 5, 2, 6, 3, 4, 3, 1, 9, 11, 12, 2, 8, 8, 12, 10,
  8, 2, 2, 5, 7, 4, 6, 5, 9, 7, 4, 12, 10, 5, 11, 9, 2, 8, 9, 7, 3, 10, 5, 1, 5, 5, 4, 6,
  5, 4, 10, 4, 1, 6, 5, 3, 3, 2, 4, 10, 7, 2, 11, 6, 5, 7, 4, 5, 11, 8, 4, 11, 7, 3, 4, 2,
  3, 8, 9, 1, 11, 10, 12, 9, 2, 10, 1, 3, 5, 9, 4, 4, 3, 7, 4, 5, 9, 9, 11, 6, 10, 3, 12,
  1, 5, 10, 5, 8, 5, 8, 6, 9, 4, 2, 7, 5, 7, 1, 6, 2, 6, 4, 3, 6, 11, 11, 7, 4, 7, 10, 6,
  1, 10, 8, 6, 11, 10, 9, 1, 3, 3, 3, 9, 10, 2, 7, 1, 2, 6, 6, 3, 9, 11, 8, 12, 9, 5, 5,
  1, 11, 6, 6, 3, 9, 9, 12, 7, 4, 10, 12, 5, 11, 6, 4, 5, 11, 12, 1, 11, 12, 6, 1, 4, 3,
  1, 4, 5, 6, 5, 11, 1, 10, 10, 10, 8, 6, 8, 8, 2, 3, 1, 4, 5, 4, 1, 4, 3, 11, 8, 3, 11,
  11, 12, 11, 10, 11, 10, 2, 12, 6, 1, 3, 4, 10, 11, 8, 7, 11, 4, 5, 10, 8, 2, 4, 12, 1,
  1, 12, 6, 9, 12, 8, 8, 12, 12, 12, 4, 6, 4, 11, 9, 3, 11, 1, 10, 11, 4, 10, 9, 1, 1, 8,
  4, 8, 8, 7, 4, 1, 4, 1, 3, 12, 6, 1, 5, 8, 2, 8, 2, 10, 1, 3, 10, 12, 11, 5, 5, 6, 5, 4,
  3, 8, 1, 1, 6, 1, 5, 6, 4, 7, 10, 5, 11, 1, 4, 7, 7, 2, 5, 10, 11, 10, 9, 9, 4, 3, 5, 5,
  10, 6, 6, 4, 11, 5, 8, 9, 7, 1, 2, 9, 2, 12, 12, 5, 2, 11, 1, 7, 7, 8, 1, 7, 7, 3, 4, 5,
  5, 2, 5, 11, 11, 4, 9, 7, 6, 8, 8, 8, 8, 12, 11, 4, 5, 4, 9, 2, 8, 10, 9, 7, 12, 3, 1, 6,
  4, 4, 5, 12, 2, 8, 5, 12, 6, 12, 11, 9, 12, 2, 6, 11, 1, 5, 2, 12, 3, 10, 2, 9, 10, 2,
  10, 9, 8, 5, 9, 3, 7, 1, 6, 7, 8, 12, 11, 1, 7, 9, 1, 12, 11, 12, 6, 1, 7, 6, 4, 4, 3,
  11, 7, 5, 8, 4, 2, 8, 6, 1, 8, 5, 5, 6, 2, 6, 12, 5, 4, 2, 1, 11, 12, 2, 11, 5, 8, 12,
  12, 4, 5, 6, 8, 8, 4, 2, 2, 11, 10, 11, 10, 1, 9, 2, 10, 9, 5, 9, 3, 12, 4, 9, 5, 12, 6,
  5, 8, 12, 7, 7, 3, 12, 1, 4, 2, 4, 9, 8, 2, 3, 11, 9, 8, 12, 11, 5, 10, 12, 2, 2, 4, 8,
  1, 3, 5, 7, 8, 4, 8, 4, 5, 3, 10, 7, 8, 9, 8, 10, 2, 10, 8, 9, 5, 7, 6, 11, 1, 5, 10, 9,
  11, 11, 1, 3, 4, 11, 1, 6, 2, 3, 4, 7, 7, 3, 3, 10, 10, 3, 12, 10, 2, 11, 5, 11, 8, 2,
  6, 3, 3, 8, 4, 7, 4, 2, 4, 12, 6, 11, 11, 6, 10, 5, 7, 2, 1, 10, 2, 4, 9, 5, 12, 6, 3,
  3, 3, 12, 4, 6, 1, 10, 12, 4, 9, 2, 12, 3, 9, 3, 7, 6, 10, 12, 12, 6, 10, 3, 7, 8, 12,
  3, 3, 10, 7, 4, 9, 2, 6, 1, 4, 5, 6, 8, 10, 2, 8, 2, 5, 10, 2, 2, 8, 3, 7, 12, 12, 7, 6,
  9, 9, 10, 12, 1, 4, 12, 2, 5, 1, 1, 12, 11, 3, 5, 7, 1, 10, 5, 2, 4, 10, 2, 3, 12, 3, 5,
  7, 2, 11, 2, 2, 11, 3, 5, 12, 5, 9, 6, 12, 6, 10, 11, 7, 2, 2, 1, 5, 5, 10, 8, 12, 2,
  4, 9, 3, 12, 6, 8, 8, 11, 8, 5, 11, 8, 2, 6, 12, 1, 7, 4, 9, 7, 12, 3, 8, 11, 7, 6, 10,
  10, 5, 4, 8, 3, 5, 11, 6, 2, 12, 1, 4, 3, 8, 3, 6, 7, 12, 7, 11, 1, 7, 5, 3, 5, 10, 4,
  12, 12, 4, 6, 8, 2, 11, 3, 4, 3, 4, 6, 9, 11, 6, 3, 7, 12, 10, 12, 3, 9, 6, 8, 7, 12,
  4, 6, 10, 8, 4, 1, 5, 2, 2, 8, 7, 4, 5, 3, 6, 12, 7, 6, 7, 8, 10, 12, 4, 8, 9, 12, 7, 8,
  6, 12, 7, 10, 8, 2, 4, 8, 5, 9, 12, 3, 6, 8, 1, 8, 6, 8, 7, 7, 6, 6, 7, 12, 12, 7, 9,
  11, 12, 7, 5, 6, 10, 12, 12, 4, 9, 11, 1, 10, 11, 10, 10, 11, 6, 7, 5, 12, 4, 2, 6, 9,
  10, 3, 2, 8, 1, 1, 2, 12, 6, 2, 3, 12, 4, 4, 11, 9, 11, 4, 11, 6, 11, 6, 1, 7, 9, 12,
  9, 1, 7, 1, 7, 1, 12, 5, 11, 11, 12, 5, 8, 10, 11, 11, 9, 4, 3, 1, 9, 10, 5, 1, 3, 1}
```

```
In[69]:= freqs = BinCounts[dice, {1, SIDE+1, 1}]
```

```
Out[69]=
```

```
{80, 83, 76, 91, 93, 80, 77, 90, 76, 76, 82, 96}
```

```
ln[70]:= chi2 = 0; For[i = 1, i ≤ SIDE, i++,
  chi2 = chi2 + (freqs[[i]] - SIZE / SIDE)^2 / (SIZE / SIDE)
]; Print[N[chi2]]
```

6.992

```
ln[71]:= myList = {}; For[j = 1, j ≤ 1000, j++,
  dice = RandomInteger[{1, SIDE}, SIZE];
  freqs = BinCounts[dice, {1, SIDE + 1, 1}];
  chi2 = 0; For[i = 1, i ≤ SIDE, i++,
  chi2 = chi2 + (freqs[[i]] - SIZE / SIDE)^2 / (SIZE / SIDE)
]; AppendTo[myList, N[chi2]];
];
Print[myList]
```

```
{6.752, 7.568, 15.368, 11.672, 6.848, 14.792, 15.824, 13.88, 15.224, 6.08, 4.304, 11.24,
  10.688, 12.44, 8.096, 18.824, 4.568, 15.104, 14.456, 10.184, 14.384, 7.52, 14.312,
  10.64, 7.28, 16.472, 6.728, 10.928, 10.544, 15.992, 11.36, 12.368, 12.272, 18.512,
  12.896, 15.056, 14.864, 12.248, 8.864, 12.872, 12.752, 20.216, 9.368, 15.128, 10.544,
  14.792, 13.16, 14.648, 9.344, 8.888, 12.008, 13.256, 14.216, 8.096, 8.552, 14.456,
  8.096, 8.576, 11.336, 7.856, 10.832, 4.376, 10.76, 14.12, 25.592, 7.112, 5.24,
  8.096, 8.408, 14.864, 14.816, 15.944, 4.04, 6.656, 15.32, 11.24, 15.416, 10.232,
  5., 11., 15.512, 15.752, 18.776, 13.088, 4.4, 5.072, 11.792, 8.072, 9.176, 11.048,
  17.696, 3.344, 5.36, 6.944, 8.216, 14.12, 4.592, 17.408, 11.312, 9.92, 11.192, 8.432,
  11.072, 19.976, 9.848, 11.48, 7.448, 3.008, 12.08, 4.568, 12.872, 9.512, 1.904,
  13.256, 9.512, 22.424, 11.024, 5.864, 14.672, 10.04, 10.784, 18.92, 10.088, 18.56,
  7.328, 14.264, 11., 9.224, 13.448, 4.832, 18.392, 7.568, 17.744, 16.952, 18.152,
  6.104, 5.48, 12.752, 13.16, 10.088, 4.952, 7.88, 11.336, 9.896, 2.456, 9.536, 12.2,
  9.872, 20.384, 15.416, 7.52, 19.904, 7.904, 5.6, 15.152, 11.192, 5.12, 7.64, 9.2,
  4.112, 10.856, 18.848, 17.912, 7.712, 12.392, 14.648, 8.096, 16.904, 17.072, 12.464,
  6.632, 10.76, 19.184, 19.16, 20.048, 8.552, 3.56, 8.096, 12.08, 5.36, 10.376, 19.28,
  4.808, 6.464, 7.208, 12.776, 8.36, 7.904, 10.952, 8.144, 6.776, 9.944, 5.312,
  13.664, 5.672, 9.776, 7.424, 6.152, 12.8, 13.544, 12.944, 13.616, 10.256, 8.264,
  14.528, 9.608, 13.208, 6.512, 7.256, 9.776, 6.704, 5.648, 9.248, 12.008, 13.28,
  14.168, 12.2, 6.152, 16.424, 11.36, 8.072, 17.336, 5.96, 5.264, 9.32, 10.664, 5.312,
  10.136, 5.168, 7.16, 6.464, 9.248, 5.624, 8.768, 15.224, 8.648, 5.552, 13.016,
  22.352, 17.576, 8.072, 11.672, 11.456, 18.368, 12.344, 21.344, 10.016, 8.72, 3.176,
  14.12, 3.8, 12.536, 11.6, 8.312, 8.792, 3.464, 18.32, 10.856, 9.248, 7.328, 6.536,
  12.272, 9.344, 4.592, 21.776, 13.592, 15.272, 8.912, 3.152, 12.656, 7.928, 10.448,
  6.992, 4.04, 11.432, 9.512, 21.968, 12.152, 14.408, 8.816, 5.192, 10.28, 4.424,
  9.728, 10.016, 4.568, 7.616, 17.792, 15.296, 17.696, 7.352, 9.56, 5.696, 12.632,
  14.432, 10.28, 21.872, 23.072, 14.648, 6.344, 13.784, 8.504, 8.864, 13.112, 11.096,
  19.64, 12.968, 14.36, 9.008, 14.744, 6.008, 21.776, 7.64, 16.112, 9.464, 10.496,
  25.496, 7.976, 10.832, 11.816, 13.016, 10.352, 7.304, 10.712, 9.464, 13.328, 10.856,
  13.4, 18.776, 10.376, 9.464, 21.152, 10.712, 5.264, 14.888, 22.376, 6.56, 16.928,
  10.904, 13.52, 9.2, 10.832, 10.64, 14.336, 6.272, 13.472, 9.152, 9.944, 3.368,
```

7.688, 9.248, 16.064, 13.544, 20.912, 13.736, 3.992, 6.8, 8.072, 9.608, 8.072,
 15.224, 8.936, 20.936, 6.128, 6.92, 12.488, 20.336, 11.576, 15.8, 15.152, 17.432,
 15.488, 10.4, 20.456, 7.784, 7.928, 5.432, 6.416, 10.832, 9.152, 9.056, 12.56,
 10.304, 10.544, 6.752, 12.752, 11.192, 6.728, 11.504, 23., 9.92, 7.472, 13.232,
 8.912, 12.704, 10.544, 14.72, 12.608, 8.168, 13.88, 14.168, 6.344, 10.112, 4.52,
 8.984, 7.952, 9.68, 4.424, 5.792, 5.432, 7.328, 7.424, 11.408, 7.112, 3.584, 9.608,
 14.576, 19.832, 8.264, 10.952, 5.72, 9.776, 13.88, 15.752, 15.968, 15.152, 16.376,
 10.448, 11.792, 9.92, 13.88, 8.696, 4.736, 6.392, 9.44, 3.56, 8.888, 5.144, 12.128,
 14.648, 5.936, 8.864, 25.616, 14.384, 10.712, 12.752, 6.968, 6.92, 5.888, 8.792,
 3.632, 20.432, 6.128, 8.672, 8.072, 9.632, 14.576, 10.136, 13.856, 7.88, 10.256,
 13.208, 10.688, 16.544, 10.616, 12.608, 10.136, 12.248, 9.44, 7.64, 18.536, 8.6,
 8.48, 14.456, 14.72, 25.88, 12.008, 7.76, 10.088, 15.656, 8., 9.872, 17., 13.52,
 16.088, 8.648, 7.616, 16.328, 19.592, 7.496, 9.44, 8.072, 14.816, 7.184, 14.168,
 18.128, 14.864, 10.688, 8.576, 6.824, 21.416, 10.544, 6.056, 7.256, 11.696, 12.584,
 9.32, 12.536, 19.208, 12.896, 17.768, 21.176, 9.416, 12.464, 2.72, 10.664, 5.288,
 6.152, 10.4, 12.392, 9.32, 11.744, 12.872, 9.464, 11.12, 8.552, 9.584, 6.464, 4.112,
 7.52, 10.184, 4.904, 6.776, 18.68, 11.84, 12.584, 5.24, 10.736, 9.296, 7.832, 6.128,
 8.936, 12.896, 16.52, 7.784, 10.856, 11.816, 13.472, 10.472, 12.248, 7.928, 12.104,
 6.968, 8.912, 16.688, 5.288, 6.152, 7.352, 13.496, 9.224, 11., 5.84, 8.072, 7.112,
 9.824, 5.84, 6.728, 16.016, 12.944, 9.152, 10.928, 17.672, 9.464, 13.904, 8.336,
 7.04, 13.112, 15.512, 14.216, 6.416, 18.656, 5.576, 16.064, 18.848, 4.616, 3.32,
 7.448, 14.696, 4.52, 12.992, 12.704, 14.888, 12.08, 14.624, 5.648, 7.784, 14.648,
 6.704, 14.048, 17.792, 8.528, 9.632, 7.88, 12.104, 17.936, 11.72, 14.624, 15.296,
 10.448, 19.544, 19.952, 5.384, 13.88, 5.624, 7.4, 6.152, 5.24, 15.536, 9.44, 19.904,
 10.328, 7.808, 5.432, 3.2, 4.208, 11.768, 11.528, 11.504, 10.472, 11.36, 14.528,
 10.376, 8.96, 18.152, 11.888, 9.704, 10.784, 13.112, 14.552, 16.808, 19.832, 6.368,
 11.096, 14.84, 9.896, 8.456, 9.416, 12.728, 6.416, 9.68, 6.176, 8.768, 13.808,
 6.512, 13.664, 5.48, 10.016, 11.336, 15.08, 5.672, 11.048, 13.472, 2.672, 14.384,
 9.008, 10.352, 11.984, 3.608, 13.592, 13.784, 7.136, 11.168, 14.456, 18.416, 5.96,
 7.88, 8.144, 6.968, 12.176, 8.96, 7.784, 3.632, 4.736, 12.152, 4.016, 9.68, 14.624,
 27.416, 9.176, 3.296, 13.112, 12.944, 6.368, 7.52, 14.936, 9.128, 17.144, 11.024,
 14.456, 9.2, 6.224, 11.432, 18.392, 11.264, 10.88, 6.536, 9.2, 9.128, 9.92, 6.488,
 7.928, 19.952, 11.624, 8.408, 11.096, 3.32, 13.16, 9.056, 28.016, 6.944, 11.864,
 6.008, 5.648, 11.384, 9.2, 7.064, 9.776, 14.576, 11.336, 7.928, 9.608, 13.064,
 10.352, 2.816, 10.496, 17.36, 12.008, 6.56, 11.84, 10.16, 16.544, 8.816, 7.736,
 17.432, 15.872, 7.256, 13.4, 6.824, 14.744, 7.304, 12.392, 8.936, 17.192, 15.728,
 9.464, 10.328, 25.712, 8.528, 3.368, 15.416, 10.64, 5.336, 7.088, 13.16, 9.464,
 8.024, 8.36, 11.624, 7.544, 15.344, 3.752, 8.888, 13.28, 10.376, 6.224, 9.392,
 11.6, 9.536, 19.328, 11.24, 9.776, 12.656, 9.224, 11.576, 21.416, 7.904, 8.84,
 12.32, 8.384, 6.32, 5.792, 12.824, 16.184, 10.064, 11.72, 15.752, 8.192, 10.184,
 3.152, 7.472, 20.432, 9.44, 19.808, 7.376, 15.584, 6.488, 5.648, 5.576, 8.72, 9.536,
 4.808, 10.784, 4.64, 18.488, 8.408, 8.24, 13.304, 11.744, 6.416, 13.616, 8.528,
 9.656, 13.544, 7.76, 13.208, 14.576, 14.504, 9.176, 4.04, 18.296, 8.96, 5.96, 7.64,
 16.472, 13.496, 7.28, 5.672, 17.672, 8.96, 14.36, 10.352, 12.608, 14.912, 7.208,
 13.928, 10.232, 9.248, 30.632, 15.632, 6.536, 8.408, 10.76, 12.752, 19.544, 11.768,

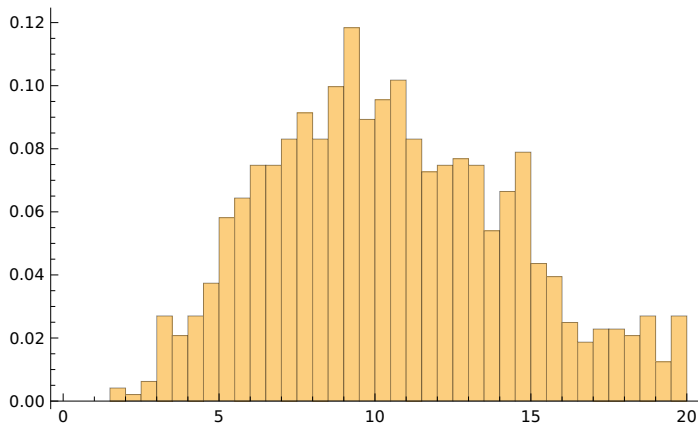
```

16.928, 12.896, 13.04, 13.616, 7.784, 5.504, 8.84, 11.576, 5.048, 10.352, 9.488,
7.112, 12.248, 8.744, 6.992, 6.584, 8.72, 3.152, 9.224, 13.328, 12.152, 14.432,
9.08, 9.488, 5.912, 9.728, 18.824, 20.984, 5.048, 9.44, 6.608, 7.088, 15.776,
13.832, 7.256, 7.4, 14.576, 13.304, 9.488, 14.024, 14.648, 5.24, 6.176, 14.216,
4.112, 9.872, 7.16, 13.04, 14.48, 19.544, 9.872, 14.192, 12.176, 9.44, 9.752,
10.736, 9.536, 6.296, 11.288, 24.032, 12.872, 7.688, 4.736, 10.712, 5.792, 20.6,
8.24, 11.528, 3.656, 10.616, 12.32, 6.056, 8.84, 9.296, 16.376, 7.424, 7.976, 12.44,
9.2, 23.216, 7.208, 11.12, 11.792, 20.936, 15.344, 17.096, 19.256, 10.568, 5.744,
14.912, 17.048, 8.792, 14.48, 5.96, 10.424, 14.768, 11.264, 10.52, 12.464, 18.56,
11.6, 8.288, 8.696, 12.896, 15.8, 5.84, 10.952, 8.36, 11.648, 5., 12.656, 6.608,
7.976, 10.04, 10.952, 11.504, 8.864, 11.456, 11.792, 6.968, 6.704, 10.952, 10.976,
15.872, 11.984, 7.736, 14.48, 13.976, 4.736, 6.248, 11.048, 8.456, 8.816, 9.656,
13.256, 9.344, 20.768, 10.76, 10.184, 10.928, 13.112, 11.312, 1.736, 14.216, 7.112}

```

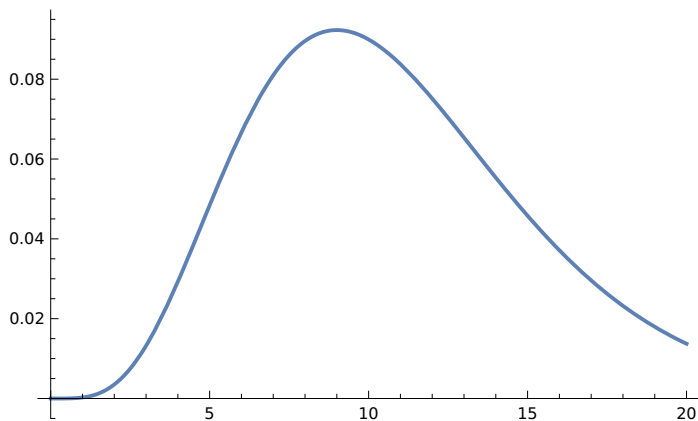
```
In[72]:= h1 = Histogram[myList, {0, 20, 0.5}, "PDF"]
```

Out[72]=



```
In[73]:= h2 = Plot[PDF[ChiSquareDistribution[SIDE - 1], x], {x, 0, 20}]
```

Out[73]=



In[74]:= Show[h1, h2]

Out[74]=

