Based on
The Langlands Program - Numberphile
https://www.youtube.com/watch?v=4dyytPboqvE

In[1]:=
```
p = 5
```

Out[1]= 5

In[2]:=
```
For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
Print[" x= ", x, " y = ", y, " ", " left: ", y^2 + y, " right: ", x^3 - x^2]
]
]
```

```
 x= 0 y = 0  left: 0 right: 0
 x= 0 y = 1  left: 2 right: 0
 x= 0 y = 2  left: 6 right: 0
 x= 0 y = 3  left: 12 right: 0
 x= 0 y = 4  left: 20 right: 0
 x= 1 y = 0  left: 0 right: 0
 x= 1 y = 1  left: 2 right: 0
 x= 1 y = 2  left: 6 right: 0
 x= 1 y = 3  left: 12 right: 0
 x= 1 y = 4  left: 20 right: 0
 x= 2 y = 0  left: 0 right: 4
 x= 2 y = 1  left: 2 right: 4
 x= 2 y = 2  left: 6 right: 4
 x= 2 y = 3  left: 12 right: 4
 x= 2 y = 4  left: 20 right: 4
 x= 3 y = 0  left: 0 right: 18
 x= 3 y = 1  left: 2 right: 18
 x= 3 y = 2  left: 6 right: 18
 x= 3 y = 3  left: 12 right: 18
 x= 3 y = 4  left: 20 right: 18
 x= 4 y = 0  left: 0 right: 48
 x= 4 y = 1  left: 2 right: 48
 x= 4 y = 2  left: 6 right: 48
 x= 4 y = 3  left: 12 right: 48
 x= 4 y = 4  left: 20 right: 48
```

In[3]:=
```
For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
Print[" x= ", x, " y = ", y, " ",
   " left: ",  Mod[y^2 + y, p], " right: ", Mod[x^3 - x^2, p]]
]
]
```

```
 x= 0 y = 0  left: 0 right: 0
 x= 0 y = 1  left: 2 right: 0
 x= 0 y = 2  left: 1 right: 0
 x= 0 y = 3  left: 2 right: 0
 x= 0 y = 4  left: 0 right: 0
 x= 1 y = 0  left: 0 right: 0
 x= 1 y = 1  left: 2 right: 0
 x= 1 y = 2  left: 1 right: 0
 x= 1 y = 3  left: 2 right: 0
 x= 1 y = 4  left: 0 right: 0
 x= 2 y = 0  left: 0 right: 4
 x= 2 y = 1  left: 2 right: 4
 x= 2 y = 2  left: 1 right: 4
 x= 2 y = 3  left: 2 right: 4
 x= 2 y = 4  left: 0 right: 4
 x= 3 y = 0  left: 0 right: 3
 x= 3 y = 1  left: 2 right: 3
 x= 3 y = 2  left: 1 right: 3
 x= 3 y = 3  left: 2 right: 3
 x= 3 y = 4  left: 0 right: 3
 x= 4 y = 0  left: 0 right: 3
 x= 4 y = 1  left: 2 right: 3
 x= 4 y = 2  left: 1 right: 3
 x= 4 y = 3  left: 2 right: 3
 x= 4 y = 4  left: 0 right: 3
```

In[8]:=
```
For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
left = Mod[y^2 + y, p];
right = Mod[x^3 - x^2, p];
If[left == right,
Print[" x= ", x, " y = ", y, " ",
    " left: ",  Mod[y^2 + y, p], " right: ", Mod[x^3 - x^2, p]],
]
]
]
```

```
 x= 0 y = 0  left: 0 right: 0
 x= 0 y = 4  left: 0 right: 0
 x= 1 y = 0  left: 0 right: 0
 x= 1 y = 4  left: 0 right: 0
```

In[4]:=
```
SolCount = 0; For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
left = Mod[y^2 + y, p];
right = Mod[x^3 - x^2, p];
If[left == right,
(*Print[" x= ", x, " y = ", y, " ",
    " left: ",  Mod[y^2+y,p]," right: ", Mod[x^3-x^2,p]] *) SolCount++,
]
]
]; Print["Number of solutions: ", SolCount]
```

```
Number of solutions: 4
```

In[5]:=
```
For[p = 2, p < 50, p++,
If[PrimeQ[p], Print[p],]
](* end loop over p *)
```

```
2
3
5
7
11
13
17
19
23
29
31
37
41
43
47
```

In[6]:=

```
For[p = 2, p < 50, p++,
If[PrimeQ[p],
SolCount = 0; For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
left = Mod[y^2 + y, p];
right = Mod[x^3 - x^2, p];
If[left == right,
(*Print[" x= ", x, " y = ", y, " ",
       " left: ",  Mod[y^2+y,p]," right: ", Mod[x^3-x^2,p]] *) SolCount++,
](* end if solution  *)
](* end y loop *)
](* end x loop*);
Print["Number of solutions: ", SolCount]
,](*end if a prime *)
](* end loop over p *)
```

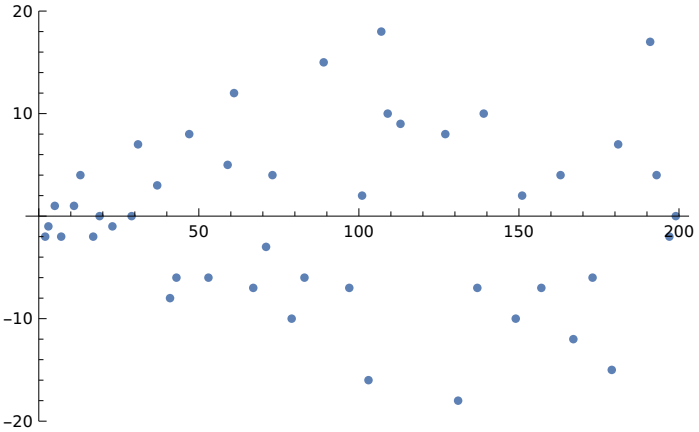
```
Number of solutions: 4
Number of solutions: 4
Number of solutions: 4
Number of solutions: 9
Number of solutions: 10
Number of solutions: 9
Number of solutions: 19
Number of solutions: 19
Number of solutions: 24
Number of solutions: 29
Number of solutions: 24
Number of solutions: 34
Number of solutions: 49
Number of solutions: 49
Number of solutions: 39
```

In[7]:=
```
APS = {};
For[p = 2, p < 200, p++,
If[PrimeQ[p],
SolCount = 0; For[x = 0, x < p, x++,
For[y = 0, y < p, y++,
left = Mod[y^2 + y, p];
right = Mod[x^3 - x^2, p];
If[left == right,
(*Print[" x= ", x, " y = ", y, " ",
       " left: ",  Mod[y^2+y,p]," right: ", Mod[x^3-x^2,p]] *) SolCount++,
](* end if solution  *)
](* end y loop *)
](* end x loop*);
mytuple = {};
AppendTo[mytuple, p];
AppendTo[mytuple, p - SolCount];
AppendTo[APS, mytuple];
,](*end if a prime *)
]; (* end loop over p *)Print[APS]
```

```
{{2, -2}, {3, -1}, {5, 1}, {7, -2}, {11, 1}, {13, 4}, {17, -2}, {19, 0}, {23, -1}, {29, 0}, {31, 7},
 {37, 3}, {41, -8}, {43, -6}, {47, 8}, {53, -6}, {59, 5}, {61, 12}, {67, -7}, {71, -3}, {73, 4},
 {79, -10}, {83, -6}, {89, 15}, {97, -7}, {101, 2}, {103, -16}, {107, 18}, {109, 10}, {113, 9},
 {127, 8}, {131, -18}, {137, -7}, {139, 10}, {149, -10}, {151, 2}, {157, -7}, {163, 4},
 {167, -12}, {173, -6}, {179, -15}, {181, 7}, {191, 17}, {193, 4}, {197, -2}, {199, 0}}
```

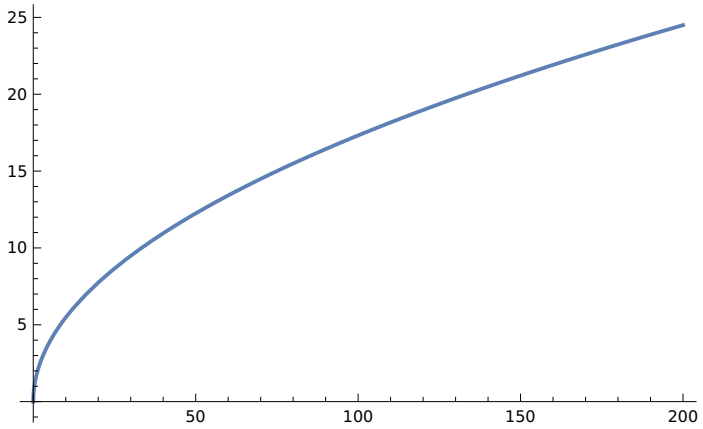
In[9]:=
```
apsPlot = ListPlot[APS]
```

Out[9]=



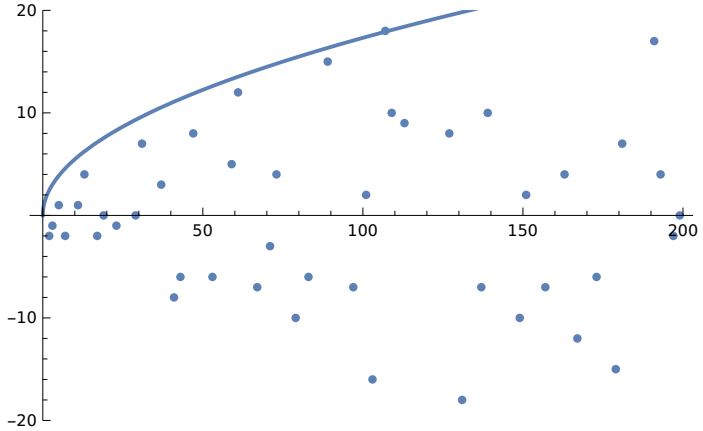
In[10]:=
```
sqrt1 = Plot[Sqrt[3 * x], {x, 0, 200}]
```

Out[10]=



In[11]:=
```
Show[apsPlot, sqrt1]
```

Out[11]=



In[12]:=
```
H[q_] := q * Product[(1 - q^i)^2 * (1 - q^(i * 11))^2, {i, 1, Infinity}]
```

In[13]:=
```
HExp = Series[H[q], {q, 0, 200}]
```

Out[13]=

$$q - 2q^2 - q^3 + 2q^4 + q^5 + 2q^6 - 2q^7 - 2q^9 - 2q^{10} + q^{11} - 2q^{12} + 4q^{13} + 4q^{14} - q^{15} - 4q^{16} - 2q^{17} + 4q^{18} + 2q^{20} + 2q^{21} - 2q^{22} - q^{23} - 4q^{25} - 8q^{26} + 5q^{27} - 4q^{28} + 2q^{30} + 7q^{31} + 8q^{32} - q^{33} + 4q^{34} - 2q^{35} - 4q^{36} + 3q^{37} - 4q^{39} - 8q^{41} - 4q^{42} - 6q^{43} + 2q^{44} - 2q^{45} + 2q^{46} + 8q^{47} + 4q^{48} - 3q^{49} + 8q^{50} + 2q^{51} + 8q^{52} - 6q^{53} - 10q^{54} + q^{55} + 5q^{59} - 2q^{60} + 12q^{61} - 14q^{62} + 4q^{63} - 8q^{64} + 4q^{65} + 2q^{66} - 7q^{67} - 4q^{68} + q^{69} + 4q^{70} - 3q^{71} + 4q^{73} - 6q^{74} + 4q^{75} - 2q^{77} + 8q^{78} - 10q^{79} - 4q^{80} + q^{81} + 16q^{82} - 6q^{83} + 4q^{84} - 2q^{85} + 12q^{86} + 15q^{89} + 4q^{90} - 8q^{91} - 2q^{92} - 7q^{93} - 16q^{94} - 8q^{96} - 7q^{97} + 6q^{98} - 2q^{99} - 8q^{100} + 2q^{101} - 4q^{102} - 16q^{103} + 2q^{105} + 12q^{106} + 18q^{107} + 10q^{108} + 10q^{109} - 2q^{110} - 3q^{111} + 8q^{112} + 9q^{113} - q^{115} - 8q^{117} - 10q^{118} + 4q^{119} + q^{121} - 24q^{122} + 8q^{123} + 14q^{124} - 9q^{125} - 8q^{126} + 8q^{127} + 6q^{129} - 8q^{130} - 18q^{131} - 2q^{132} + 14q^{134} + 5q^{135} - 7q^{137} - 2q^{138} + 10q^{139} - 4q^{140} - 8q^{141} + 6q^{142} + 4q^{143} + 8q^{144} - 8q^{146} + 3q^{147} + 6q^{148} - 10q^{149} - 8q^{150} + 2q^{151} + 4q^{153} + 4q^{154} + 7q^{155} - 8q^{156} - 7q^{157} + 20q^{158} + 6q^{159} + 8q^{160} + 2q^{161} - 2q^{162} + 4q^{163} - 16q^{164} - q^{165} + 12q^{166} - 12q^{167} + 3q^{169} + 4q^{170} - 12q^{172} - 6q^{173} + 8q^{175} - 4q^{176} - 5q^{177} - 30q^{178} - 15q^{179} - 4q^{180} + 7q^{181} + 16q^{182} - 12q^{183} + 3q^{185} + 14q^{186} - 2q^{187} + 16q^{188} - 10q^{189} + 17q^{191} + 8q^{192} + 4q^{193} + 14q^{194} - 4q^{195} - 6q^{196} - 2q^{197} + 4q^{198} + O[q]^{201}$$

In[15]:=
```
SeriesCoefficient[HExp, {q, 0, 113}]
```

Out[15]=

9

In[16]:=
```
APS2 = {};
For[p = 2, p < 200, p++,
If[PrimeQ[p],
mytuple = {};
AppendTo[mytuple, p];
AppendTo[mytuple, SeriesCoefficient[HExp, {q, 0, p}]];
AppendTo[APS2, mytuple];
](* end if *)
](*end loop over p *); Print[APS2]
```

{{2, -2}, {3, -1}, {5, 1}, {7, -2}, {11, 1}, {13, 4}, {17, -2}, {19, 0}, {23, -1}, {29, 0}, {31, 7}, {37, 3}, {41, -8}, {43, -6}, {47, 8}, {53, -6}, {59, 5}, {61, 12}, {67, -7}, {71, -3}, {73, 4}, {79, -10}, {83, -6}, {89, 15}, {97, -7}, {101, 2}, {103, -16}, {107, 18}, {109, 10}, {113, 9}, {127, 8}, {131, -18}, {137, -7}, {139, 10}, {149, -10}, {151, 2}, {157, -7}, {163, 4}, {167, -12}, {173, -6}, {179, -15}, {181, 7}, {191, 17}, {193, 4}, {197, -2}, {199, 0}}

In[18]:=
```
APS
```

Out[18]=

{{2, -2}, {3, -1}, {5, 1}, {7, -2}, {11, 1}, {13, 4}, {17, -2}, {19, 0}, {23, -1}, {29, 0}, {31, 7}, {37, 3}, {41, -8}, {43, -6}, {47, 8}, {53, -6}, {59, 5}, {61, 12}, {67, -7}, {71, -3}, {73, 4}, {79, -10}, {83, -6}, {89, 15}, {97, -7}, {101, 2}, {103, -16}, {107, 18}, {109, 10}, {113, 9}, {127, 8}, {131, -18}, {137, -7}, {139, 10}, {149, -10}, {151, 2}, {157, -7}, {163, 4}, {167, -12}, {173, -6}, {179, -15}, {181, 7}, {191, 17}, {193, 4}, {197, -2}, {199, 0}}