Playing around with the geometric distribution. If p is the probability that a certain outcome occurs (such as rolling "snake eyes"), then p*(1-p)^n is the probability that the outcome occurs on the (n+1)th for THE FIRST TIME in a sequence of independent events (such repeatedly rolling two dice).

Probabilities should add to one

In[1]:=
```
Sum[p*(1 - p)^n, {n, 0, Infinity}]
```

Out[1]= 1

The sum of x^n from n=0 to infinity is 1/(1-x) -- the geometric series. Sums with factors of n inside the sum can be obtained by applying x (d/dx) to the sum. So the following are straightforward -- though tedious (with Mathematica). "cm" is for "central moment" cm2 is the variance. cm3 and cm4 are related to the skewness and kurtosis respectively.

In[38]:=
```
mu = Sum[p*(1 - p)^n (n + 1), {n, 0, Infinity}]
```

Out[38]=

$$\frac{1}{p}$$

In[4]:=
```
cm2 = Sum[p*(1 - p)^n (n + 1 - 1/p)^2, {n, 0, Infinity}]
```

Out[4]= $$\frac{1-p}{p^2}$$

In[5]:=
```
cm3 = Sum[p*(1 - p)^n (n + 1 - 1/p)^3, {n, 0, Infinity}]
```

Out[5]= $$\frac{2-3p+p^2}{p^3}$$

In[7]:=
```
cm4 = Sum[p*(1 - p)^n (n + 1 - 1/p)^4, {n, 0, Infinity}]
```

Out[7]= $$\frac{9-18p+10p^2-p^3}{p^4}$$

In[9]:=
```
sk = Simplify[cm3/cm2^(3/2)]
```

Out[9]= $$\frac{2-p}{\sqrt{\frac{1-p}{p^2}}\; p}$$

In[10]:=
```
ku = Simplify[cm4/cm2^2]
```

Out[10]=

$$\frac{9-9p+p^2}{1-p}$$

The median is the value halfway through the values of a distribution when the values are ordered. If we take a finite sum of the probabilities and find when that reaches 1/2, we should be at the expected median

In[103]:=

```
Solve[Sum[p * (1 - p) ^ n, {n, 0, M - 1}] == 1 / 2, M]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[103]=

$$\left\{\left\{M \to -\frac{\text{Log}[2]}{\text{Log}[1-p]}\right\}\right\}$$

Compare with some simulations for rolling two dice until we roll a snake eyes (two ones -- sum of two). The corresponding p is 1/36

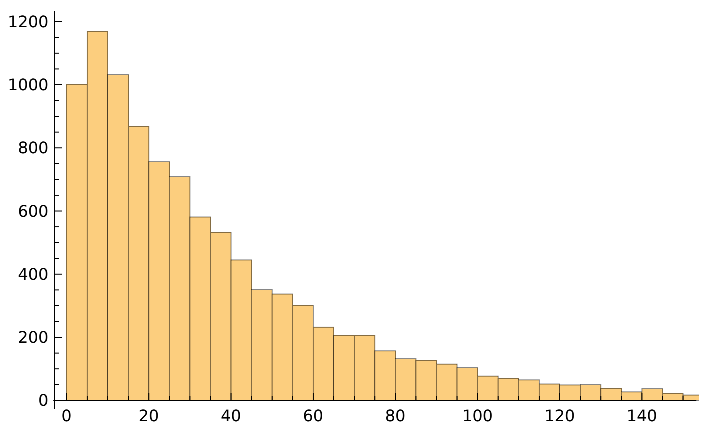
In[110]:=

```
SampleNum = 10 000;
Samples = {};
For[i = 1, i ≤ SampleNum, i++,
MySum = 0;
MyCount = 0;
While[MySum ≠ 2,
Die1 = RandomInteger[{1, 6}]; Die2 = RandomInteger[{1, 6}];
MySum = Die1 + Die2;
MyCount++]; (*end while loop when roll "snake eyes" *)
AppendTo[Samples, MyCount];
](* end loop over samples*);
```

In[113]:=

```
Histogram[Samples]
```

Out[113]=



In[114]:=

```
N[Mean[Samples]]
```

Out[114]=

36.5695

In[115]:=

```
p = 1 / 36
```

Out[115]=

$$\frac{1}{36}$$

In[116]:=

```
N[mu]
```

Out[116]=

```
36.
```

In[117]:=

```
N[Variance[Samples]]
```

Out[117]=

```
1298.49
```

In[118]:=

```
N[cm2]
```

Out[118]=

```
1260.
```

In[119]:=

```
N[Skewness[Samples]]
```

Out[119]=

```
2.0445
```

In[120]:=

```
N[sk]
```

Out[120]=

```
2.0002
```

In[121]:=

```
N[Kurtosis[Samples]]
```

Out[121]=

```
9.4234
```

In[122]:=

```
N[ku]
```

Out[122]=

```
9.00079
```

In[123]:=

```
Median[Samples]
```

Out[123]=

```
26
```

In[124]:=

```
N[Log[0.5] / Log[1 - p]]
```

Out[124]=

```
24.6051
```

In[125]:=

```
Clear[p]
```

In[126]:=

```
mu
```

Out[126]=

$$\frac{1}{p}$$

The sampling distributions -- by Central Limit Theorem should be normal

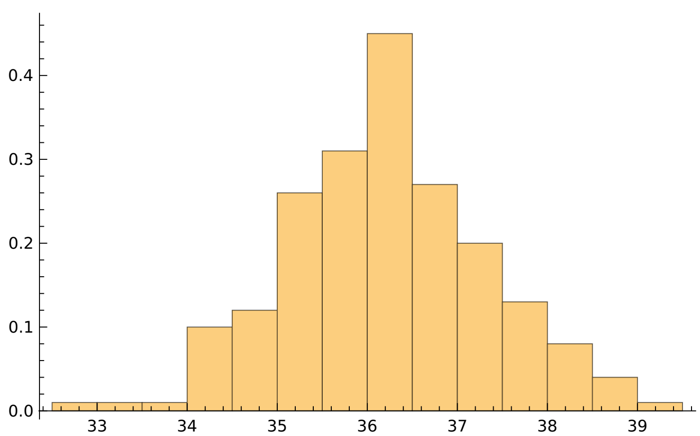
In[127]:=

```
MeanSamples = {}; VarianceSamples = {}; SkewSamples = {}; KurtSamples = {};
For[j = 1, j ≤ 200, j++,
SampleNum = 1000;
Samples = {};
For[i = 1, i ≤ SampleNum, i++,
MySum = 0;
MyCount = 0;
While[MySum ≠ 2,
Die1 = RandomInteger[{1, 6}]; Die2 = RandomInteger[{1, 6}];
MySum = Die1 + Die2;
MyCount++]; (*end while loop when roll "snake eyes" *)
AppendTo[Samples, MyCount];
](* end loop over samples*);
AppendTo[MeanSamples, N[Mean[Samples]]];
AppendTo[VarianceSamples, N[Variance[Samples]]];
AppendTo[SkewSamples, N[Skewness[Samples]]];
AppendTo[KurtSamples, N[Kurtosis[Samples]]]
]; (* Print[KurtSamples] *)
```

In[129]:=

```
Histogram[MeanSamples, Automatic, "PDF"]
```

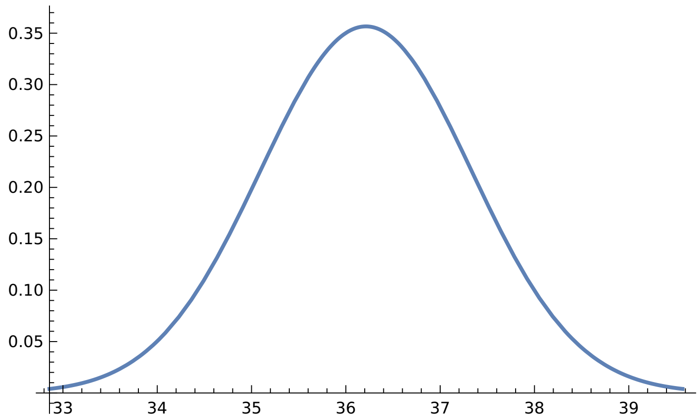
Out[129]=



In[130]:=

```
Plot[PDF[NormalDistribution[Mean[MeanSamples], StandardDeviation[MeanSamples]], x],
{x, Mean[MeanSamples] - 3 * StandardDeviation[MeanSamples],
  Mean[MeanSamples] + 3 * StandardDeviation[MeanSamples]}]
```

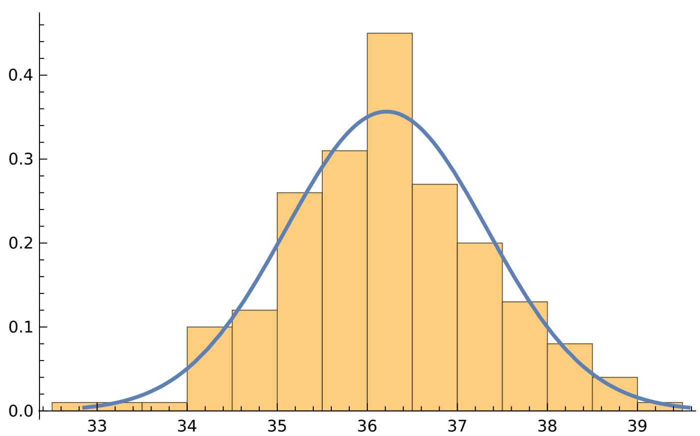
Out[130]=



In[131]:=

```
Show[Histogram[MeanSamples, Automatic, "PDF"],
 Plot[PDF[NormalDistribution[Mean[MeanSamples], StandardDeviation[MeanSamples]], x],
{x, Mean[MeanSamples] - 3 * StandardDeviation[MeanSamples],
   Mean[MeanSamples] + 3 * StandardDeviation[MeanSamples]}]]
```

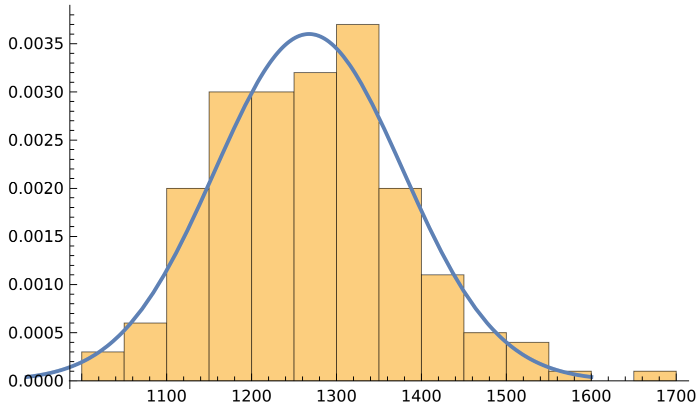
Out[131]=



In[132]:=

```
Show[Histogram[VarianceSamples, Automatic, "PDF"], Plot[
  PDF[NormalDistribution[Mean[VarianceSamples], StandardDeviation[VarianceSamples]], x],
{x, Mean[VarianceSamples] - 3 * StandardDeviation[VarianceSamples],
   Mean[VarianceSamples] + 3 * StandardDeviation[VarianceSamples]}]]
```

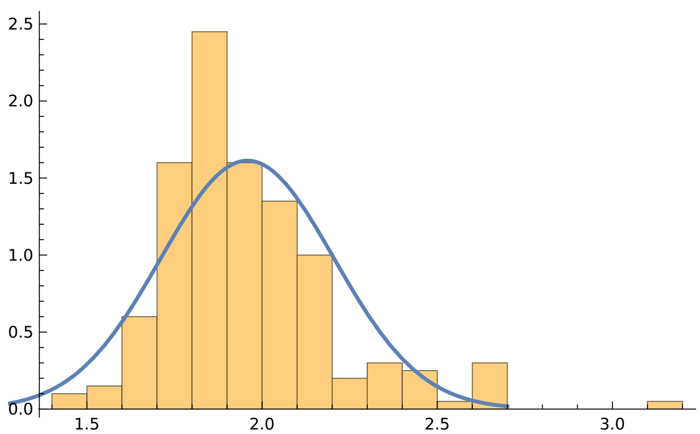
Out[132]=



In[133]:=

```
Show[Histogram[SkewSamples, Automatic, "PDF"],
 Plot[PDF[NormalDistribution[Mean[SkewSamples], StandardDeviation[SkewSamples]], x],
{x, Mean[SkewSamples] - 3 * StandardDeviation[SkewSamples],
   Mean[SkewSamples] + 3 * StandardDeviation[SkewSamples]}]]
```

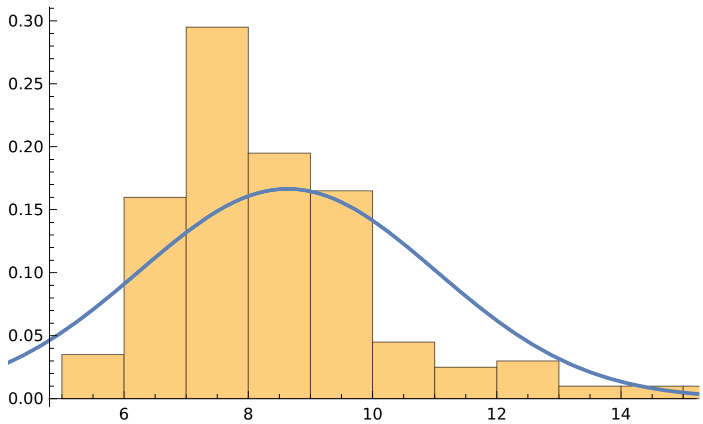
Out[133]=



In[134]:=

```
Show[Histogram[KurtSamples, Automatic, "PDF"],
 Plot[PDF[NormalDistribution[Mean[KurtSamples], StandardDeviation[KurtSamples]], x],
{x, Mean[KurtSamples] - 3 * StandardDeviation[KurtSamples],
    Mean[KurtSamples] + 3 * StandardDeviation[KurtSamples]}]]
```

Out[134]=



In[135]:=

```
Mean[KurtSamples]
```

Out[135]=

```
8.63384
```

In[136]:=

```
StandardDeviation[KurtSamples]
```

Out[136]=

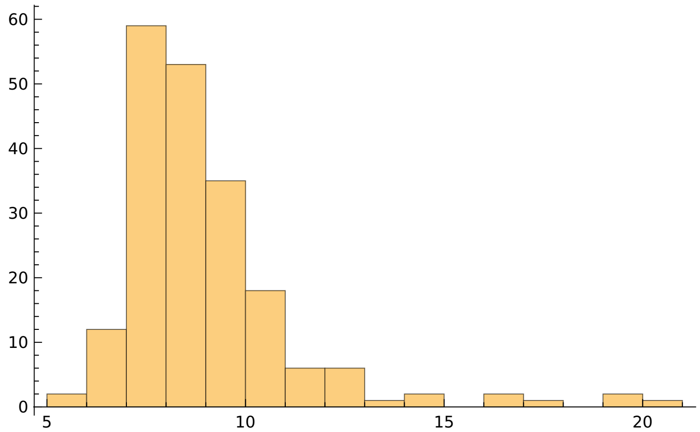
```
2.39534
```

In[80]:=

```
KurtSamples = {};
For[j = 1, j ≤ 200, j++,
SampleNum = 2000;
Samples = {};
For[i = 1, i ≤ SampleNum, i++,
MySum = 0;
MyCount = 0;
While[MySum ≠ 2,
Die1 = RandomInteger[{1, 6}]; Die2 = RandomInteger[{1, 6}];
MySum = Die1 + Die2;
MyCount++]; (*end while loop when roll "snake eyes" *)
AppendTo[Samples, MyCount];
](* end loop over samples*);
AppendTo[KurtSamples, N[Kurtosis[Samples]]]
]; (* Print[KurtSamples] *)
```

In[82]:= `Histogram[KurtSamples]`

Out[82]=



In[83]:= `Mean[KurtSamples]`

Out[83]=

9.04047

In[84]:= `StandardDeviation[KurtSamples]`

Out[84]=

2.25835