Finding the "inverted circles" for circles inscribed in a honeycomb lattice

https://en.wikipedia.org/wiki/Inversive_geometry

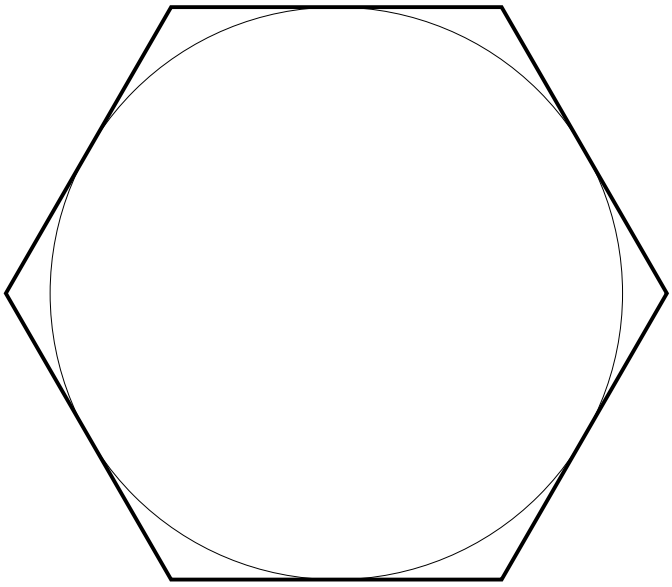
Variation on:
Infinitely Many Touching Circles - Numberphile
https://www.youtube.com/watch?v=hSsRcpIsunk

Epic Circles - Numberphile
https://www.youtube.com/watch?v=sG_6nlMZ8f4

The "length" parameter in the RegularPolygon method is the distance from center to corner
If the inscribed circle has a radius of 1, then the hexagon has a length parameter of 2/Sqrt[3]

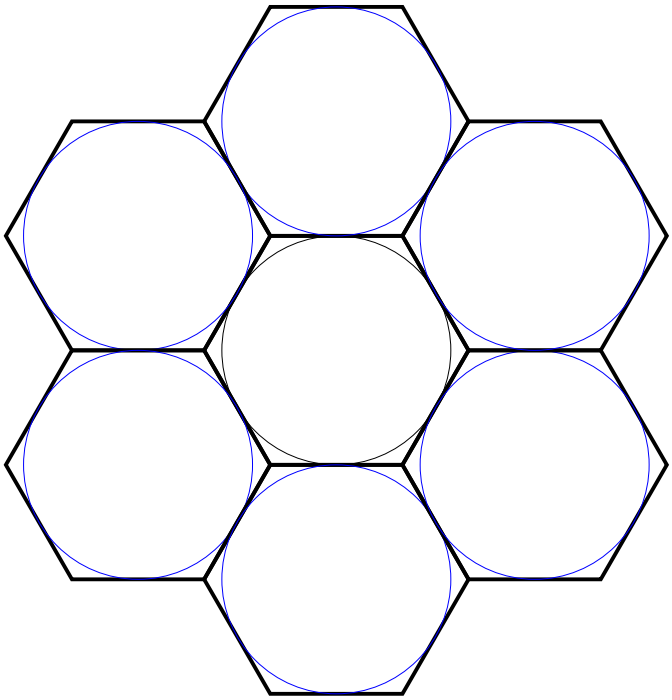
In[1]:=
```
Show[Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}],
 Graphics[Circle[{0, 0}, 1]]]
```

Out[1]=



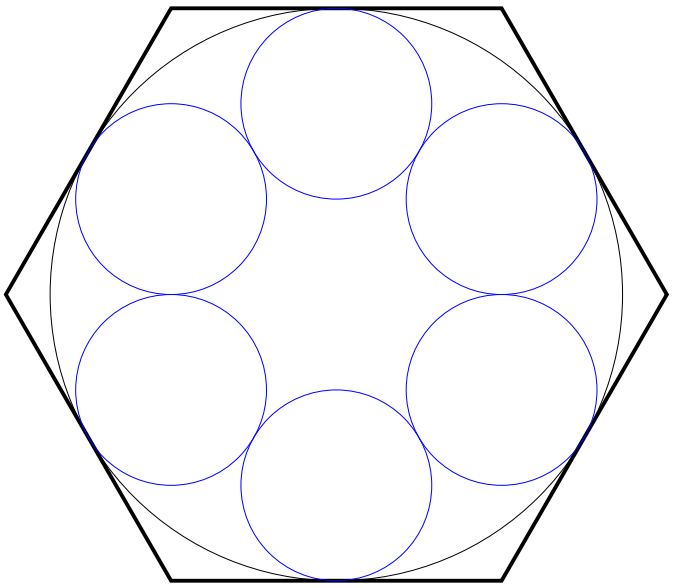
In[8]:=
```
HC = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[HC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[HC, center];
For[i = 0, i < 6, i++,
(* NN - nearest neighbors *)
NN = Graphics[{EdgeForm[Thick], Opacity[0],
    RegularPolygon[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NN];
NN = Graphics[{Blue, Circle[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 1]}];
AppendTo[HC, NN];
]; Show[HC]
```

Out[13]=



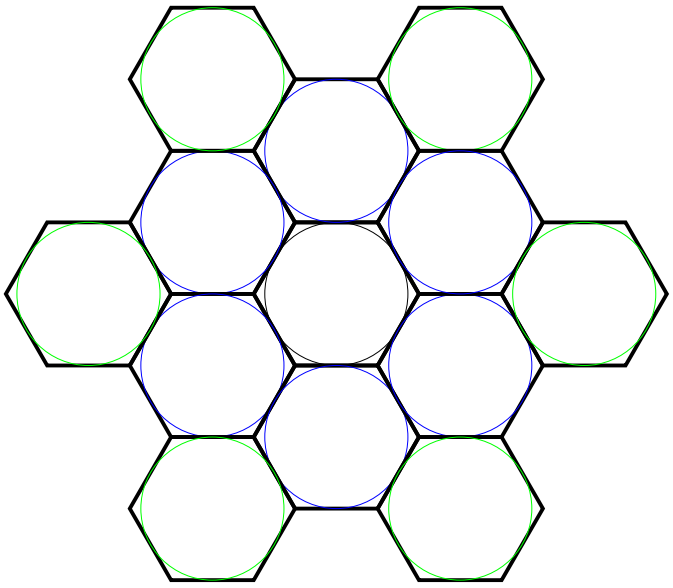
In[2]:=
```
IHC = {}; (* inverted honeycomb*)
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[IHC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[IHC, center];
For[i = 0, i < 6, i++,
(* inverted nearest neighbor circles radii  1/3  *)
NN = Graphics[{Blue, Circle[{2 / 3 * Cos[Pi / 2 + i * Pi / 3], 2 / 3 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 3]}];
AppendTo[IHC, NN];
]; Show[IHC]
```

Out[7]=



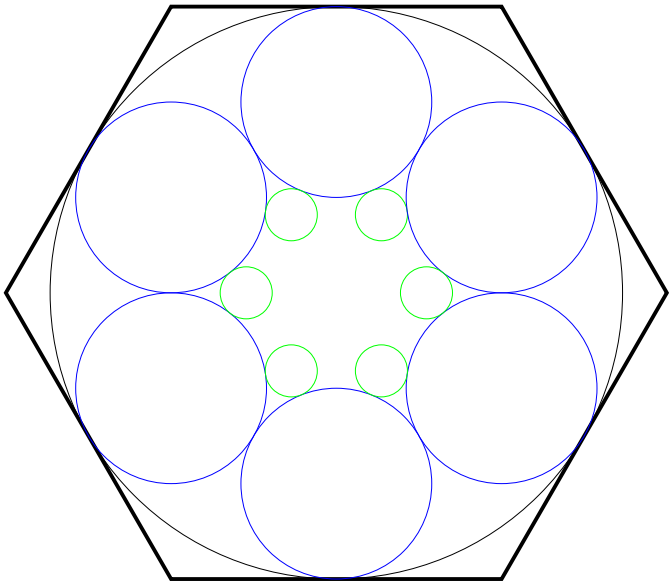
In[32]:=
```
HC = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[HC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[HC, center];
For[i = 0, i < 6, i++,
(* nearest neighbors*)
NN = Graphics[{EdgeForm[Thick], Opacity[0],
    RegularPolygon[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NN];
NN = Graphics[{Blue, Circle[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 1]}];
AppendTo[HC, NN];
(* next nearest neighbors *)
NNN = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[
     {6 / Sqrt[3] * Cos[Pi / 3 + i * Pi / 3], 6 / Sqrt[3] * Sin[Pi / 3 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NNN];
NNN = Graphics[
   {Green, Circle[{6 / Sqrt[3] * Cos[Pi / 3 + i * Pi / 3], 6 / Sqrt[3] * Sin[Pi / 3 + i * Pi / 3]}, 1]}];
AppendTo[HC, NNN];
]; Show[HC]
```

Out[37]=



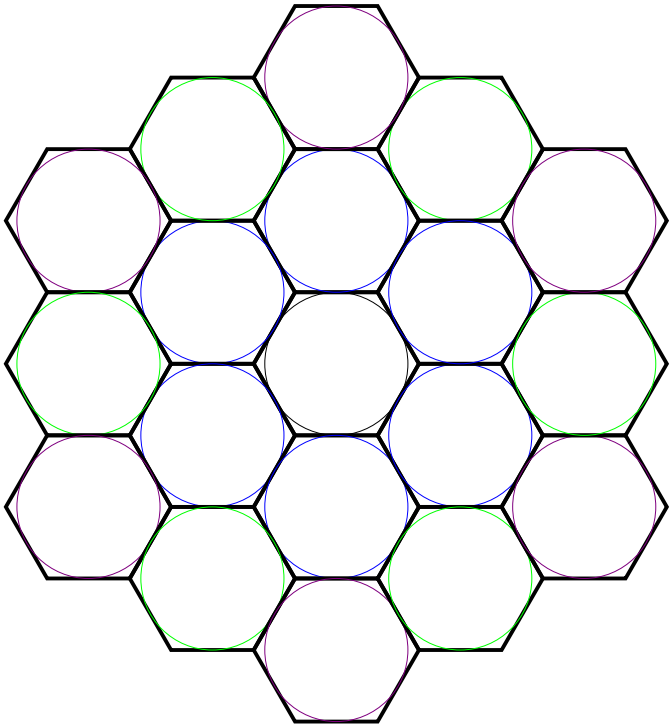
In[8]:=
```
IHC = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[IHC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[IHC, center];
For[i = 0, i < 6, i++,
(* inverted nearest neighbors radii 1/3 *)
NN = Graphics[{Blue, Circle[{2 / 3 * Cos[Pi / 2 + i * Pi / 3], 2 / 3 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 3]}];
AppendTo[IHC, NN];
(* inverted next nearest neighbors radii 1/11 *)
NNN = Graphics[{Green,
    Circle[{6 Sqrt[3] / 33 * Cos[Pi / 3 + i * Pi / 3], 6 Sqrt[3] / 33 * Sin[Pi / 3 + i * Pi / 3]}, 1 / 11]}];
AppendTo[IHC, NNN];
]; Show[IHC]
```

Out[13]=



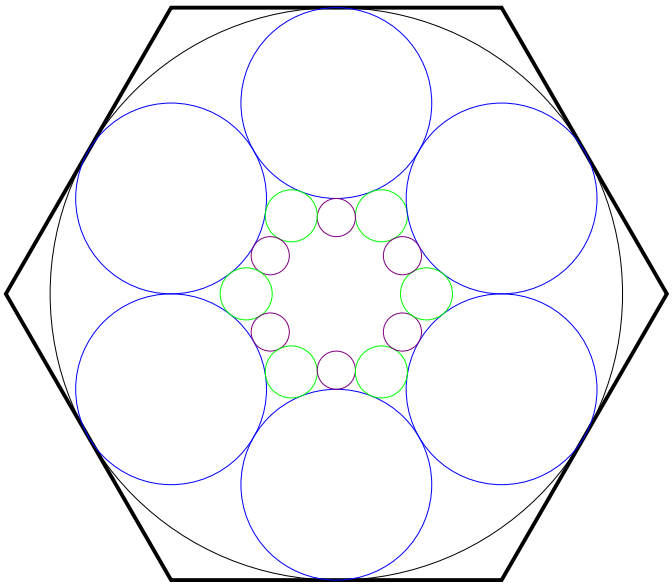
In[14]:=
```
HC = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[HC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[HC, center];
For[i = 0, i < 6, i++,
(* nearest neighbors*)
NN = Graphics[{EdgeForm[Thick], Opacity[0],
    RegularPolygon[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NN];
NN = Graphics[{Blue, Circle[{2 * Cos[Pi / 2 + i * Pi / 3], 2 * Sin[Pi / 2 + i * Pi / 3]}, 1]}];
AppendTo[HC, NN];
(* next nearest neighbors *)
NNN = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[
     {6 / Sqrt[3] * Cos[Pi / 3 + i * Pi / 3], 6 / Sqrt[3] * Sin[Pi / 3 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NNN];
NNN = Graphics[
   {Green, Circle[{6 / Sqrt[3] * Cos[Pi / 3 + i * Pi / 3], 6 / Sqrt[3] * Sin[Pi / 3 + i * Pi / 3]}, 1]}];
AppendTo[HC, NNN];
(* next-next nearest neighbors*)
NNNN = Graphics[{EdgeForm[Thick], Opacity[0],
    RegularPolygon[{4 * Cos[Pi / 2 + i * Pi / 3], 4 * Sin[Pi / 2 + i * Pi / 3]}, 2 / Sqrt[3], 6]}];
AppendTo[HC, NNNN];
NNNN = Graphics[{Purple, Circle[{4 * Cos[Pi / 2 + i * Pi / 3], 4 * Sin[Pi / 2 + i * Pi / 3]}, 1]}];
AppendTo[HC, NNNN];
]; Show[HC]
```

Out[19]=



In[20]:=
```
IHC = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[IHC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[IHC, center];
For[i = 0, i < 6, i++,
(* inverted nearest neighbors radii 1/3 *)
NN = Graphics[{Blue, Circle[{2 / 3 * Cos[Pi / 2 + i * Pi / 3], 2 / 3 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 3]}];
AppendTo[IHC, NN];
(* inverted next nearest neighbors radii 1/11*)
NNN = Graphics[{Green,
    Circle[{6 Sqrt[3] / 33 * Cos[Pi / 3 + i * Pi / 3], 6 Sqrt[3] / 33 * Sin[Pi / 3 + i * Pi / 3]}, 1 / 11]}];
AppendTo[IHC, NNN];
(* inverted next-next nearest neighbors radii 1/15 *)
NN =
  Graphics[{Purple, Circle[{4 / 15 * Cos[Pi / 2 + i * Pi / 3], 4 / 15 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 15]}];
AppendTo[IHC, NN];
]; Show[IHC]
```

Out[25]=



Next, let us invert the nearest-neighbor hexagons.

Outer radial straight-line segments invert to inner radial straight-line segments

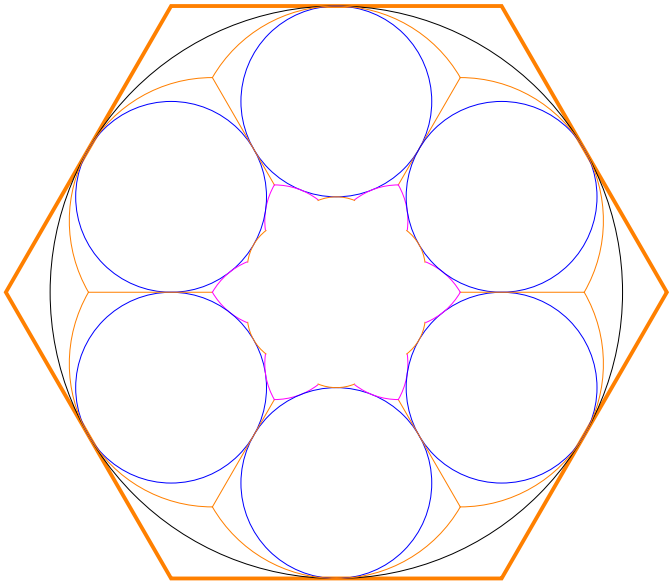
Non-radial outer straight-line segments map to arcs of interior circles that pass through the origin

In[26]:=
```
IHC = {}; (* inverted honeycomb*)
center =
  Graphics[{EdgeForm[{Orange, Thick}], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[IHC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[IHC, center];
For[i = 0, i < 6, i++,
(* inverted nearest neighbor circles radii  1/3  *)
NN = Graphics[{Blue, Circle[{2 / 3 * Cos[Pi / 2 + i * Pi / 3], 2 / 3 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 3]}];
AppendTo[IHC, NN];
(* inverting the bounding/adjacent hexagon lines *)
NN = Graphics[{Orange, Circle[{1 / 2 * Cos[Pi / 2 + i * Pi / 3], 1 / 2 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 2,
{Pi / 2 - Pi / 3 + i * Pi / 3, Pi / 2 + Pi / 3 + i * Pi / 3}]}];
AppendTo[IHC, NN];
(* inverting radial lines of the nearest neighbor hexagons *)
NN = Graphics[{Orange, Line[{{Sqrt[3] / 4 * Cos[Pi / 3 + i * Pi / 3], Sqrt[3] / 4 * Sin[Pi / 3 + i * Pi / 3]},
{Sqrt[3] / 2 * Cos[Pi / 3 + i * Pi / 3], Sqrt[3] / 2 * Sin[Pi / 3 + i * Pi / 3]}}]}];
AppendTo[IHC, NN];

(* inverting far side of nearest neighbor hexagons -- angle range ???*)
NN = Graphics[{Orange, Circle[{1 / 6 * Cos[Pi / 2 + i * Pi / 3], 1 / 6 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 6,
{Pi / 2 - Pi / 8 + i * Pi / 3, Pi / 2 + Pi / 8 + i * Pi / 3}]}];
AppendTo[IHC, NN];

(* others  angle range?? *)
NN = Graphics[{Magenta, Circle[{1 / 4 * Cos[Pi / 2 + i * Pi / 3], 1 / 4 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 4,
{-Pi / 24 + i * Pi / 3, Pi / 6 + i * Pi / 3}]}];
AppendTo[IHC, NN];
NN = Graphics[{Magenta, Circle[{1 / 4 * Cos[Pi / 2 + i * Pi / 3], 1 / 4 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 4,
{5 Pi / 6 + 5 Pi / 24 + i * Pi / 3, 5 Pi / 6 + i * Pi / 3}]}];
AppendTo[IHC, NN];
]; Show[IHC]
```

Out[31]=



In[32]:=
```
IHC = {}; (* inverted honeycomb*)
center =
  Graphics[{EdgeForm[{Orange, Thick}], Opacity[0], RegularPolygon[{0, 0}, 2 / Sqrt[3], 6]}];
AppendTo[IHC, center];
center = Graphics[Circle[{0, 0}, 1]] ;
AppendTo[IHC, center];
For[i = 0, i < 6, i++,
(* inverted nearest neighbor circles radii  1/3  *)
NN = Graphics[{Blue, Circle[{2 / 3 * Cos[Pi / 2 + i * Pi / 3], 2 / 3 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 3]}];
AppendTo[IHC, NN];

(* inverted next nearest neighbors radii 1/11*)
NNN = Graphics[{Green,
    Circle[{6 Sqrt[3] / 33 * Cos[Pi / 3 + i * Pi / 3], 6 Sqrt[3] / 33 * Sin[Pi / 3 + i * Pi / 3]}, 1 / 11]}];
AppendTo[IHC, NNN];
(* inverted next-next nearest neighbors radii 1/15 *)
NN =
  Graphics[{Purple, Circle[{4 / 15 * Cos[Pi / 2 + i * Pi / 3], 4 / 15 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 15]}];
AppendTo[IHC, NN];


(* inverting the bounding hexagon lines *)
NN = Graphics[{Orange, Circle[{1 / 2 * Cos[Pi / 2 + i * Pi / 3], 1 / 2 * Sin[Pi / 2 + i * Pi / 3]}, 1 / 2,
{Pi / 2 - Pi / 3 + i * Pi / 3, Pi / 2 + Pi / 3 + i * Pi / 3}]}];
AppendTo[IHC, NN];
(* inverting radial lines of the nearest neighbor hexagons *)
```

```
NN = Graphics[{Orange, Line[{{Sqrt[3]/4*Cos[Pi/3+i*Pi/3], Sqrt[3]/4*Sin[Pi/3+i*Pi/3]},
{Sqrt[3]/2*Cos[Pi/3+i*Pi/3], Sqrt[3]/2*Sin[Pi/3+i*Pi/3]}}]}];
AppendTo[IHC, NN];

(* inverting far side of nearest neighbor hexagons -- angle range ???*)
NN = Graphics[{Orange, Circle[{1/6*Cos[Pi/2+i*Pi/3], 1/6*Sin[Pi/2+i*Pi/3]}, 1/6,
{Pi/2-Pi/8+i*Pi/3, Pi/2+Pi/8+i*Pi/3}]}];
AppendTo[IHC, NN];

(* others  angle range?? *)
NN = Graphics[{Magenta, Circle[{1/4*Cos[Pi/2+i*Pi/3], 1/4*Sin[Pi/2+i*Pi/3]}, 1/4,
{-Pi/24+i*Pi/3, Pi/6+i*Pi/3}]}];
AppendTo[IHC, NN];
NN = Graphics[{Magenta, Circle[{1/4*Cos[Pi/2+i*Pi/3], 1/4*Sin[Pi/2+i*Pi/3]}, 1/4,
{5 Pi/6+5 Pi/24+i*Pi/3, 5 Pi/6+i*Pi/3}]}];
AppendTo[IHC, NN];
]; Show[IHC]
```

Out[37]=