In[1]:= `Exp[I * Pi]`

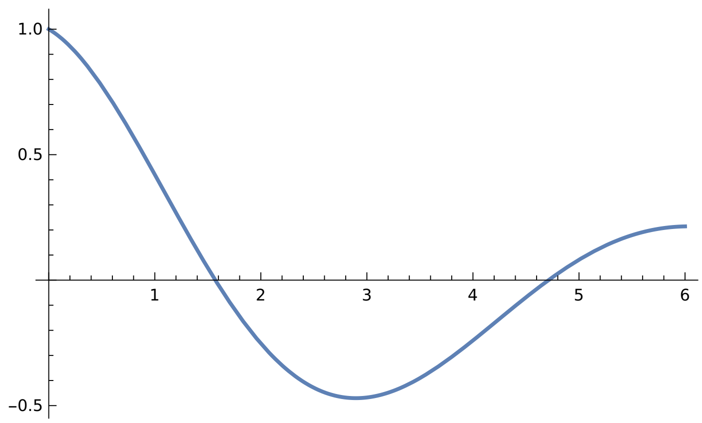
Out[1]= -1

In[2]:= `Term[g_, w_, N_, n_] := Exp[-g * n / N + I * w * n / N]`

Using Exp[i x] instead of Cos[x] to make the sum used later more obviously a geometric series.
Term is like a damped oscillator
Going to play with like a somewhat generalized weighting function or regulator as mentioned by Tony Padilla in
Does -1/12 Protect Us From Infinity? - Numberphile
https://www.youtube.com/watch?v=beakj767uG4

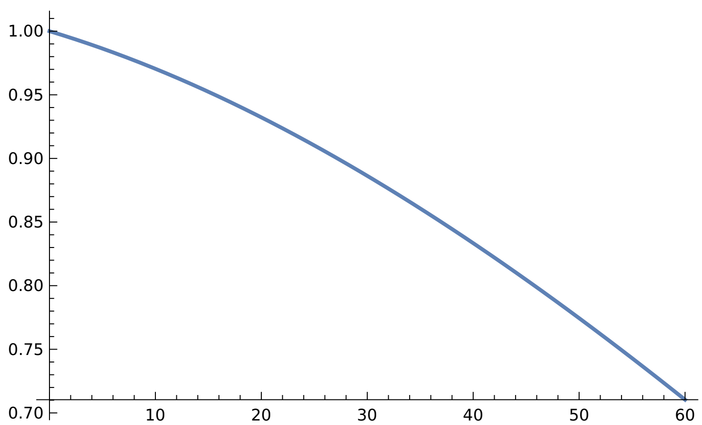
In[3]:= `Plot[Re[Term[0.25, 1, 1, n]], {n, 0, 6}]`

Out[3]=



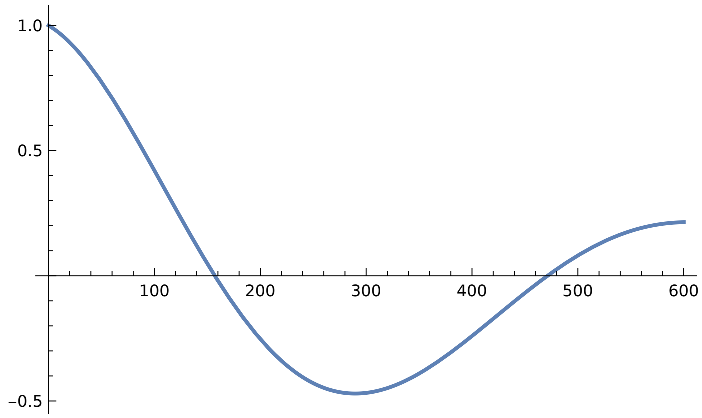
In[4]:= `Plot[Re[Term[0.25, 1, 100, n]], {n, 0, 60}]`

Out[4]=



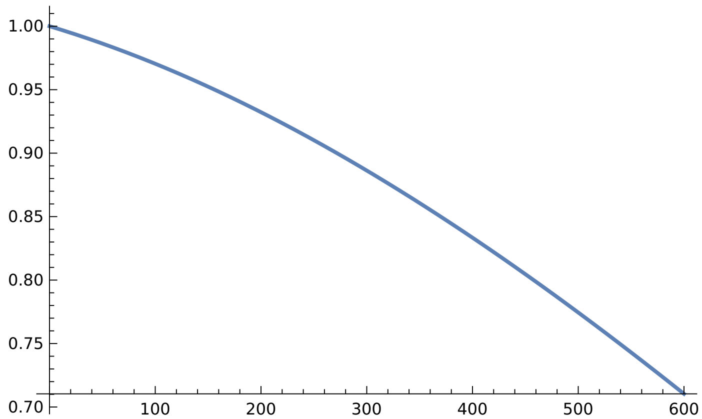
In[5]:= `Plot[Re[Term[0.25, 1, 100, n]], {n, 0, 600}]`

Out[5]=



In[6]:= `Plot[Re[Term[0.25, 1, 1000, n]], {n, 0, 600}]`

Out[6]=



In[7]:= `Geo[g_, w_, N_] := Sum[Term[g, w, N, n], {n, 0, Infinity}]`

In[8]:= `test1 = Geo[2, 3, 100]`

Out[8]= $-\frac{e^{1/50}}{e^{\frac{3 i}{100}} - e^{1/50}}$

In[9]:= `Geo2[g_, w_, N_] := 1 / (1 - Term[g, w, N, 1])`

In[10]:= `test2 = Geo2[2, 3, 100]`

Out[10]=

$$\frac{1}{1 - e^{-\frac{1}{50}+\frac{3 i}{100}}}$$

In[11]:= `Simplify[test1 - test2]`

Out[11]=

0

In[12]:=
```
Geo2[g, w, N]
```

Out[12]=

$$\frac{1}{1-e^{-\frac{g}{N}+\frac{i w}{N}}}$$

In[13]:=
```
Series[Geo2[g, w, 1/x], {x, 0, 4}]
```

Out[13]=

$$\frac{1}{(g-i w) x}+\frac{1}{2}+\frac{1}{12}(g-i w) x+\frac{1}{720}\left(-g^3+3 i g^2 w+3 g w^2-i w^3\right) x^3+O[x]^5$$

In[14]:=
```
test2a = Series[(Geo2[g, w, 1/x] + Geo2[g, -w, 1/x]) / 2, {x, 0, 4}]
```

Out[14]=

$$\frac{g}{(g-i w)(g+i w) x}+\frac{1}{2}+\frac{g x}{12}+\frac{1}{720}\left(-g^3+3 g w^2\right) x^3+O[x]^5$$

Like a series 1 + 1 + 1 + ... ???

In[15]:=
```
Simplify[test2a]
```

Out[15]=

$$\frac{g}{\left(g^2+w^2\right) x}+\frac{1}{2}+\frac{g x}{12}+\frac{1}{720}\left(-g^3+3 g w^2\right) x^3+O[x]^5$$

In[16]:=
```
-N * D[Term[g, w, N, n], g]
```

Out[16]=

$$e^{-\frac{g n}{N}+\frac{i n w}{N}} n$$

In[17]:=
```
-N * D[Geo2[g, w, N], g]
```

Out[17]=

$$\frac{e^{-\frac{g}{N}+\frac{i w}{N}}}{\left(1-e^{-\frac{g}{N}+\frac{i w}{N}}\right)^2}$$

In[18]:=
```
test3 = Series[(-(1/x) * D[Geo2[g, w, 1/x], g] + -(1/x) * D[Geo2[g, -w, 1/x], g]) / 2, {x, 0, 4}]
```

Out[18]=

$$\frac{\frac{1}{(g-i w)^2}+\frac{1}{(g+i w)^2}}{2 x^2}-\frac{1}{12}+ \cdots 4 \cdots +O[x]^5$$

Full expression not available (original memory size: 111.9 kB)

test3 is like the series 1+2+3+...

In[19]:= `Simplify[test3]`

Out[19]=

$$\frac{g^2 - w^2}{(g^2 + w^2)^2 x^2} - \frac{1}{12} + \frac{1}{240} (g^2 - w^2) x^2 + \frac{(-g^4 + 6 g^2 w^2 - w^4) x^4}{6048} + O[x]^5$$

In[20]:= `N^2*D[D[Term[g, w, N, n], g], g]`

Out[20]=

$$e^{-\frac{g n}{N} + \frac{i n w}{N}} n^2$$

In[21]:=
```
test4 =
  Series[((1/x)^2*D[D[Geo2[g, w, 1/x], g], g] + (1/x)^2*D[D[Geo2[g, -w, 1/x], g], g])/2, {x, 0, 4}]
```

Out[21]=

$$\frac{\frac{2}{(g-i w)^3} + \frac{2}{(g+i w)^3}}{2 x^3} + \frac{1}{2}\left(-\frac{1}{6} + 2\left(-\frac{1}{3} + \frac{\frac{3}{8}(g-i w)^3 - \frac{5}{4}\left(-(g-i w)^2 + 3(-g+i w)^2\right)(-g+i w) + \frac{7}{4}(g-i w)^2(-g+i w)}{3(g-i w)^3}\right) + 2\left(-\frac{1}{3} + \frac{-\frac{5}{4}\left(3(-g-i w)^2 - (g+i w)^2\right)(-g-i w) + \frac{7}{4}(-g-i w)(g+i w)^2 + \frac{3}{8}(g+i w)^3}{3(g+i w)^3}\right)\right) + \cdots 4 \cdots + O[x]^5$$

Full expression not available (original memory size: 398.5 kB)

Like the series 1+2^2+3^2 + 4^2 + 5^2 + ...

No constant term here

In[22]:= `Simplify[test4]`

Out[22]=

$$\frac{2 (g^3 - 3 g w^2)}{(g^2 + w^2)^3 x^3} - \frac{g x}{120} + \frac{(g^3 - 3 g w^2) x^3}{1512} + O[x]^5$$

In[23]:= `-N^3*D[D[D[Term[g, w, N, n], g], g], g]`

Out[23]=

$$e^{-\frac{g n}{N} + \frac{i n w}{N}} n^3$$

In[24]:=
```
test5 =
  Series[(-(1/x)^3*D[D[D[Geo2[g, w, 1/x], g], g], g] - (1/x)^3*D[D[D[Geo2[g, -w, 1/x], g], g], g])/2,
   {x, 0, 4}]
```

Out[24]=

$$\frac{\frac{6}{(g-i w)^4} + \frac{6}{(g+i w)^4}}{2 x^4} + \cdots 6 \cdots + O[x]^5$$

Full expression not available (original memory size: 1 MB)

Like the series 1+2^3+3^3+4^3+5^3 + ... ???

In[25]:= `Simplify[test5]`

Out[25]=

$$\frac{6\left(g^4-6\,g^2\,w^2+w^4\right)}{\left(g^2+w^2\right)^4\,x^4}+\frac{1}{120}+\frac{1}{504}\left(-g^2+w^2\right)x^2+\frac{\left(g^4-6\,g^2\,w^2+w^4\right)x^4}{5760}+O[x]^5$$