Take a list of 2N numbers representing a deck of cards.
Then split the array into two halves -- the first N elements in "Pile 1" and the last N elements in "Pile 2".
Interleave the piles -- first card from Pile 1 then first card from Pile 2, then second card from Pile 1, then second card from Pile 2, etc.
If we repeat that action we arrive back at the original order after some number of "perfect shuffles".
How does the number of perfect shuffles to restore order depend on N (half the size of the deck)?

https://oeis.org/A002326

In[1]:=
```
cards = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Out[1]=
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

In[2]:=
```
cards2 = {};
For[i = 1, i ≤ Length[cards], i++,
If[Mod[i, 2] == 1,
AppendTo[cards2, cards[[(i + 1) / 2]]],
AppendTo[cards2, cards[[Length[cards] / 2 + i / 2]]]
](*end if*)
](*end for loop over cards*); Print[cards2];
If[SameAs[cards][cards2], Print["YES"], Print["NO"]];
If[cards == cards2, Print["YES"], Print["NO"]];
```

```
{1, 6, 2, 7, 3, 8, 4, 9, 5, 10}
NO
NO
```

In[6]:=
```
cards = Range[10]; (* create list {1, 2, 3, ...} -- deck of cards *)
cards0 = cards; (* make copy of original list *)
ShuffleCount = 0;
cards2 = {};
While[cards0 ≠ cards2,
cards2 = {};
(* create shuffled list*)
For[i = 1, i ≤ Length[cards], i++,
If[Mod[i, 2] == 1,
AppendTo[cards2, cards[[(i + 1) / 2]]],
AppendTo[cards2, cards[[Length[cards] / 2 + i / 2]]]
](*end if*)
](*end for loop over cards*);
ShuffleCount++;
Print["Count: ", ShuffleCount, " List: ", cards2];
cards = cards2;
]
```

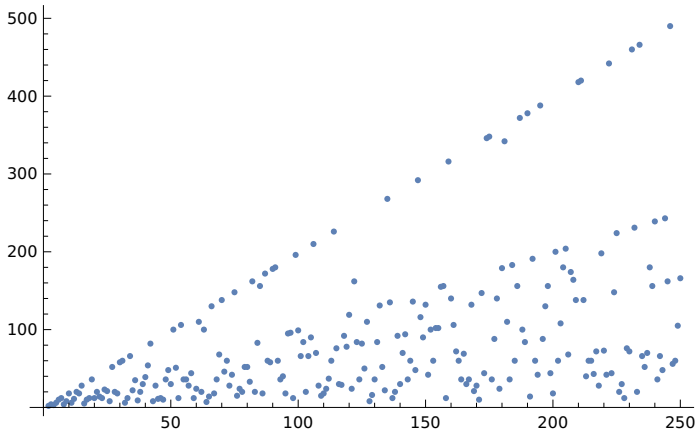
```
Count: 1 List: {1, 6, 2, 7, 3, 8, 4, 9, 5, 10}
Count: 2 List: {1, 8, 6, 4, 2, 9, 7, 5, 3, 10}
Count: 3 List: {1, 9, 8, 7, 6, 5, 4, 3, 2, 10}
Count: 4 List: {1, 5, 9, 4, 8, 3, 7, 2, 6, 10}
Count: 5 List: {1, 3, 5, 7, 9, 2, 4, 6, 8, 10}
Count: 6 List: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

In[11]:=
```
NUM = 250;
Results = {};
For[k = 2, k ≤ NUM, k++,
cards = Range[2 * k]; (* create list {1, 2, 3, ...} *)
cards0 = cards; (* make copy of original list *)
ShuffleCount = 0;
cards2 = {};
While[cards0 ≠ cards2,
cards2 = {};
(* create shuffled list*)
For[i = 1, i ≤ Length[cards], i ++,
If[Mod[i, 2] == 1,
AppendTo[cards2, cards[[(i + 1) / 2]]],
AppendTo[cards2, cards[[Length[cards] / 2 + i / 2]]]
](*end if*)
](*end for loop over cards*);
ShuffleCount++;
(*Print["Count: ",ShuffleCount, " List: ",cards2];*)
cards = cards2;
];
MyTuple = {};
AppendTo[MyTuple, k];
AppendTo[MyTuple, ShuffleCount];
AppendTo[Results, MyTuple];
](*  k loop *); Print[Results];
```

{{2, 2}, {3, 4}, {4, 3}, {5, 6}, {6, 10}, {7, 12}, {8, 4}, {9, 8}, {10, 18}, {11, 6}, {12, 11}, {13, 20},
 {14, 18}, {15, 28}, {16, 5}, {17, 10}, {18, 12}, {19, 36}, {20, 12}, {21, 20}, {22, 14}, {23, 12},
 {24, 23}, {25, 21}, {26, 8}, {27, 52}, {28, 20}, {29, 18}, {30, 58}, {31, 60}, {32, 6}, {33, 12},
 {34, 66}, {35, 22}, {36, 35}, {37, 9}, {38, 20}, {39, 30}, {40, 39}, {41, 54}, {42, 82}, {43, 8},
 {44, 28}, {45, 11}, {46, 12}, {47, 10}, {48, 36}, {49, 48}, {50, 30}, {51, 100}, {52, 51}, {53, 12},
 {54, 106}, {55, 36}, {56, 36}, {57, 28}, {58, 44}, {59, 12}, {60, 24}, {61, 110}, {62, 20},
 {63, 100}, {64, 7}, {65, 14}, {66, 130}, {67, 18}, {68, 36}, {69, 68}, {70, 138}, {71, 46},
 {72, 60}, {73, 28}, {74, 42}, {75, 148}, {76, 15}, {77, 24}, {78, 20}, {79, 52}, {80, 52},
 {81, 33}, {82, 162}, {83, 20}, {84, 83}, {85, 156}, {86, 18}, {87, 172}, {88, 60}, {89, 58},
 {90, 178}, {91, 180}, {92, 60}, {93, 36}, {94, 40}, {95, 18}, {96, 95}, {97, 96}, {98, 12},
 {99, 196}, {100, 99}, {101, 66}, {102, 84}, {103, 20}, {104, 66}, {105, 90}, {106, 210},
 {107, 70}, {108, 28}, {109, 15}, {110, 18}, {111, 24}, {112, 37}, {113, 60}, {114, 226},
 {115, 76}, {116, 30}, {117, 29}, {118, 92}, {119, 78}, {120, 119}, {121, 24}, {122, 162},
 {123, 84}, {124, 36}, {125, 82}, {126, 50}, {127, 110}, {128, 8}, {129, 16}, {130, 36},
 {131, 84}, {132, 131}, {133, 52}, {134, 22}, {135, 268}, {136, 135}, {137, 12}, {138, 20},
 {139, 92}, {140, 30}, {141, 70}, {142, 94}, {143, 36}, {144, 60}, {145, 136}, {146, 48},
 {147, 292}, {148, 116}, {149, 90}, {150, 132}, {151, 42}, {152, 100}, {153, 60}, {154, 102},
 {155, 102}, {156, 155}, {157, 156}, {158, 12}, {159, 316}, {160, 140}, {161, 106}, {162, 72},
 {163, 60}, {164, 36}, {165, 69}, {166, 30}, {167, 36}, {168, 132}, {169, 21}, {170, 28},
 {171, 10}, {172, 147}, {173, 44}, {174, 346}, {175, 348}, {176, 36}, {177, 88}, {178, 140},
 {179, 24}, {180, 179}, {181, 342}, {182, 110}, {183, 36}, {184, 183}, {185, 60}, {186, 156},
 {187, 372}, {188, 100}, {189, 84}, {190, 378}, {191, 14}, {192, 191}, {193, 60}, {194, 42},
 {195, 388}, {196, 88}, {197, 130}, {198, 156}, {199, 44}, {200, 18}, {201, 200}, {202, 60},
 {203, 108}, {204, 180}, {205, 204}, {206, 68}, {207, 174}, {208, 164}, {209, 138}, {210, 418},
 {211, 420}, {212, 138}, {213, 40}, {214, 60}, {215, 60}, {216, 43}, {217, 72}, {218, 28},
 {219, 198}, {220, 73}, {221, 42}, {222, 442}, {223, 44}, {224, 148}, {225, 224}, {226, 20},
 {227, 30}, {228, 12}, {229, 76}, {230, 72}, {231, 460}, {232, 231}, {233, 20}, {234, 466},
 {235, 66}, {236, 52}, {237, 70}, {238, 180}, {239, 156}, {240, 239}, {241, 36}, {242, 66},
 {243, 48}, {244, 243}, {245, 162}, {246, 490}, {247, 56}, {248, 60}, {249, 105}, {250, 166}}

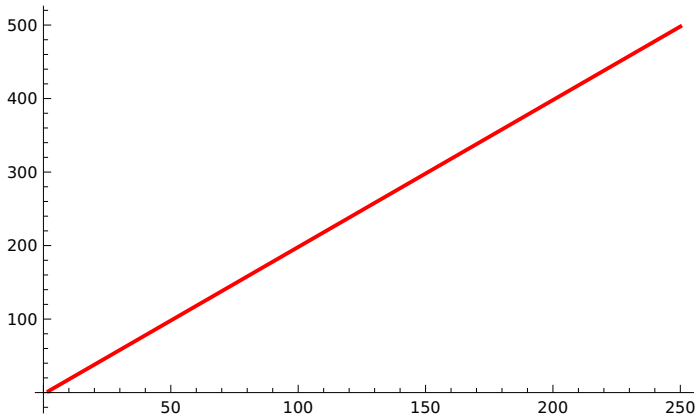
In[14]:=
```
shuffleData = ListPlot[Results]
```

Out[14]=



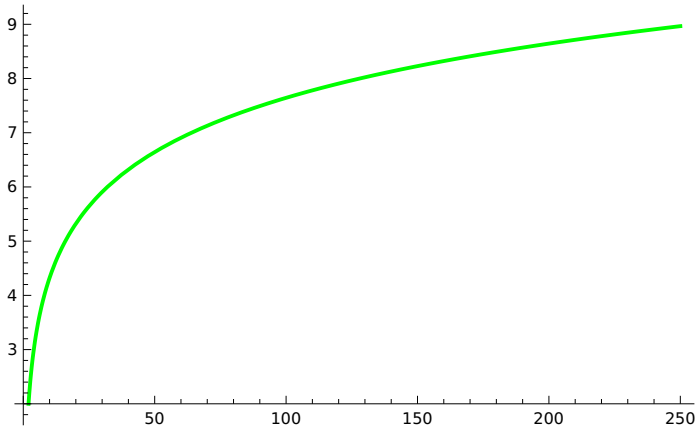
In[15]:=
```
upperBound = Plot[2 x - 2, {x, 2, NUM}, PlotStyle → {Red}]
```

Out[15]=



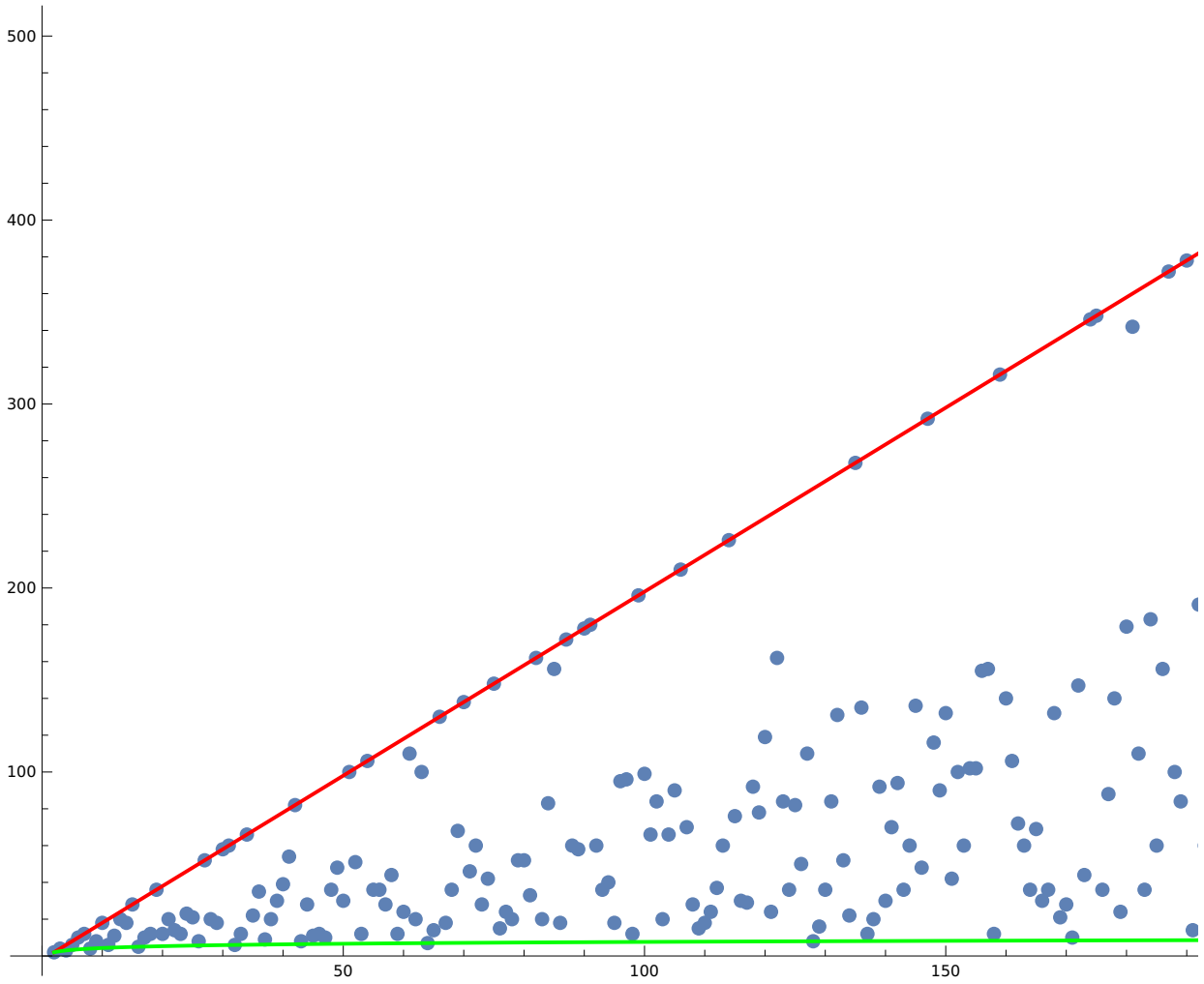
In[16]:=
```
lowerBound = Plot[Log[2, x] + 1, {x, 2, NUM}, PlotStyle → {Green}]
```

Out[16]=



In[17]:=
```
Show[shuffleData, upperBound, lowerBound]
```

Out[17]=



In[18]:=
```
uppers = Select[Results, #[[1]] * 2 - 2 == #[[2]] &]
```

Out[18]=

```
{{2, 2}, {3, 4}, {6, 10}, {7, 12}, {10, 18}, {15, 28}, {19, 36}, {27, 52}, {30, 58},
 {31, 60}, {34, 66}, {42, 82}, {51, 100}, {54, 106}, {66, 130}, {70, 138}, {75, 148},
 {82, 162}, {87, 172}, {90, 178}, {91, 180}, {99, 196}, {106, 210}, {114, 226},
 {135, 268}, {147, 292}, {159, 316}, {174, 346}, {175, 348}, {187, 372}, {190, 378},
 {195, 388}, {210, 418}, {211, 420}, {222, 442}, {231, 460}, {234, 466}, {246, 490}}
```

For the "paired consecutive" (6 and 7), (30 and 31), (90 and 91), (210 and 211) the first number seems to satisfy n*(n+1)*(n+2)*(n+3)/4

The (174 and 175) pair are not in this pattern?

In[19]:=
```
lowers = Select[Results, Log[2, #[[1]] + 1 == #[[2]] &]
```

Out[19]=

{{2, 2}, {4, 3}, {8, 4}, {16, 5}, {32, 6}, {64, 7}, {128, 8}}

The x in these {x, y} tuples are the powers of 2

The tuples immediately following those above also follow a simple pattern -- add 1 to the x above and double the y

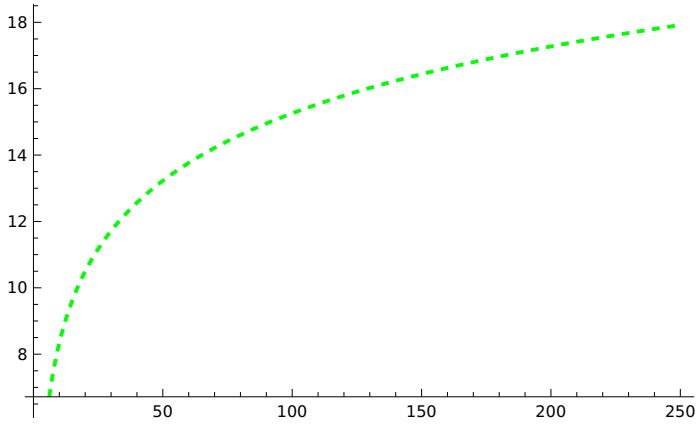
In[20]:=
```
lowers2 = Select[Results, 2 * (Log[2, #[[1]] - 1] + 1) == #[[2]] &]
```

Out[20]=

{{2, 2}, {3, 4}, {5, 6}, {9, 8}, {17, 10}, {33, 12}, {65, 14}, {129, 16}}

In[21]:=
```
lower2Plot = Plot[2 * (Log[2, x - 1] + 1), {x, 2, NUM}, PlotStyle → {Green, Dashed}]
```

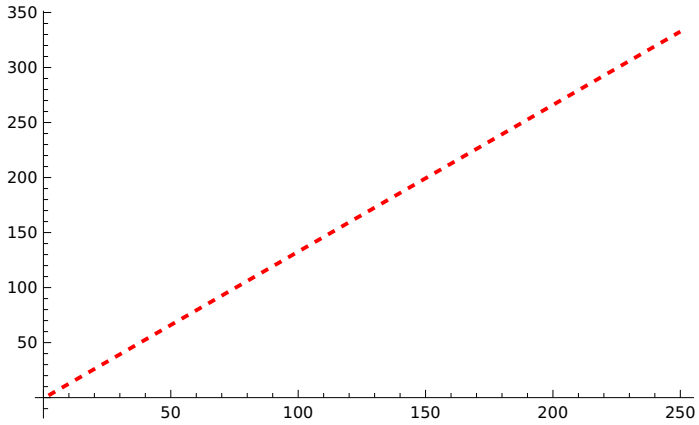
Out[21]=



Many of the tuples fall on lines with (simple ?) rational slopes

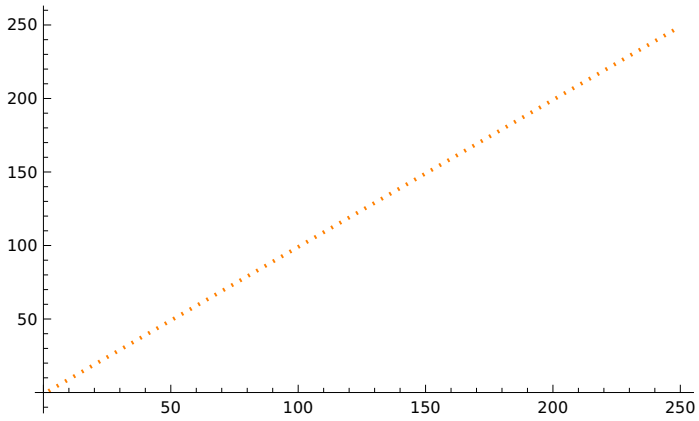
In[22]:=
```
subUpper = Plot[4 / 3 * (x + 1) - 2, {x, 2, NUM}, PlotStyle → {Red, Dashed}]
```

Out[22]=



In[23]:=
```
subUppers = Select[Results, 4 / 3 * (#[[1]] + 1) - 2 == #[[2]] &]
```
Out[23]=
```
{{2, 2}, {5, 6}, {14, 18}, {41, 54}, {122, 162}}
```

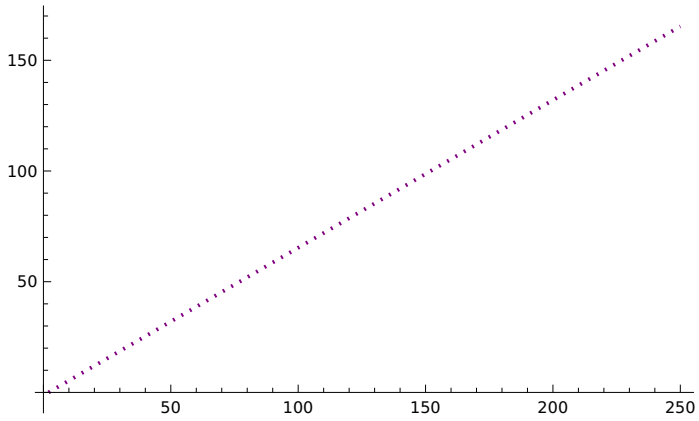
In[24]:=
```
halfUpper = Plot[x - 1, {x, 2, NUM}, PlotStyle → {Orange, Dotted}]
```
Out[24]=



In[25]:=
```
halves = Select[Results, #[[1]] * 1 - 1 == #[[2]] &]
```
Out[25]=
```
{{4, 3}, {9, 8}, {12, 11}, {21, 20}, {24, 23}, {36, 35}, {40, 39}, {49, 48},
 {52, 51}, {69, 68}, {84, 83}, {96, 95}, {97, 96}, {100, 99}, {120, 119}, {132, 131},
 {136, 135}, {156, 155}, {157, 156}, {180, 179}, {184, 183}, {192, 191},
 {201, 200}, {205, 204}, {225, 224}, {232, 231}, {240, 239}, {244, 243}}
```

In[26]:=
```
xxx = Select[Results, #[[1]] * 2 / 3 - 4 / 3 == #[[2]] &]
```
Out[26]=
```
{{8, 4}, {11, 6}, {17, 10}, {20, 12}, {29, 18}, {35, 22}, {44, 28}, {56, 36},
 {71, 46}, {80, 52}, {89, 58}, {92, 60}, {101, 66}, {107, 70}, {119, 78}, {125, 82},
 {152, 100}, {155, 102}, {161, 106}, {197, 130}, {209, 138}, {224, 148}, {245, 162}}
```

In[27]:=
```
xxxPlot = Plot[2 / 3 x - 4 / 3, {x, 2, NUM}, PlotStyle → {Purple, Dotted}]
```
Out[27]=

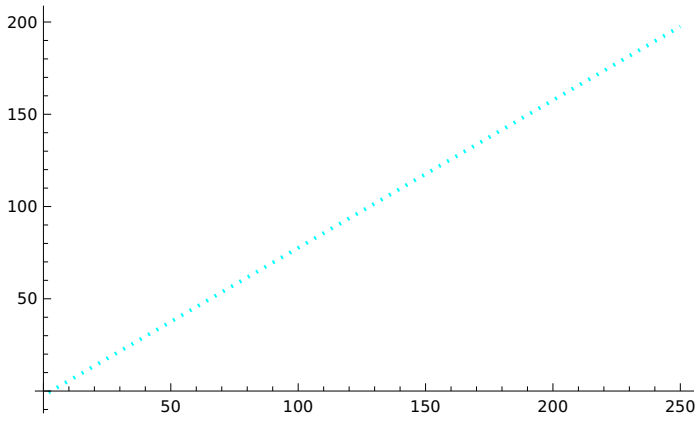


In[28]:=
```
yyy = Select[Results, #[[1]] * 4 / 5 - 12 / 5 == #[[2]] &]
```

Out[28]=
```
{{8, 4}, {18, 12}, {28, 20}, {48, 36}, {58, 44}, {118, 92},
 {148, 116}, {168, 132}, {178, 140}, {198, 156}, {208, 164}}
```

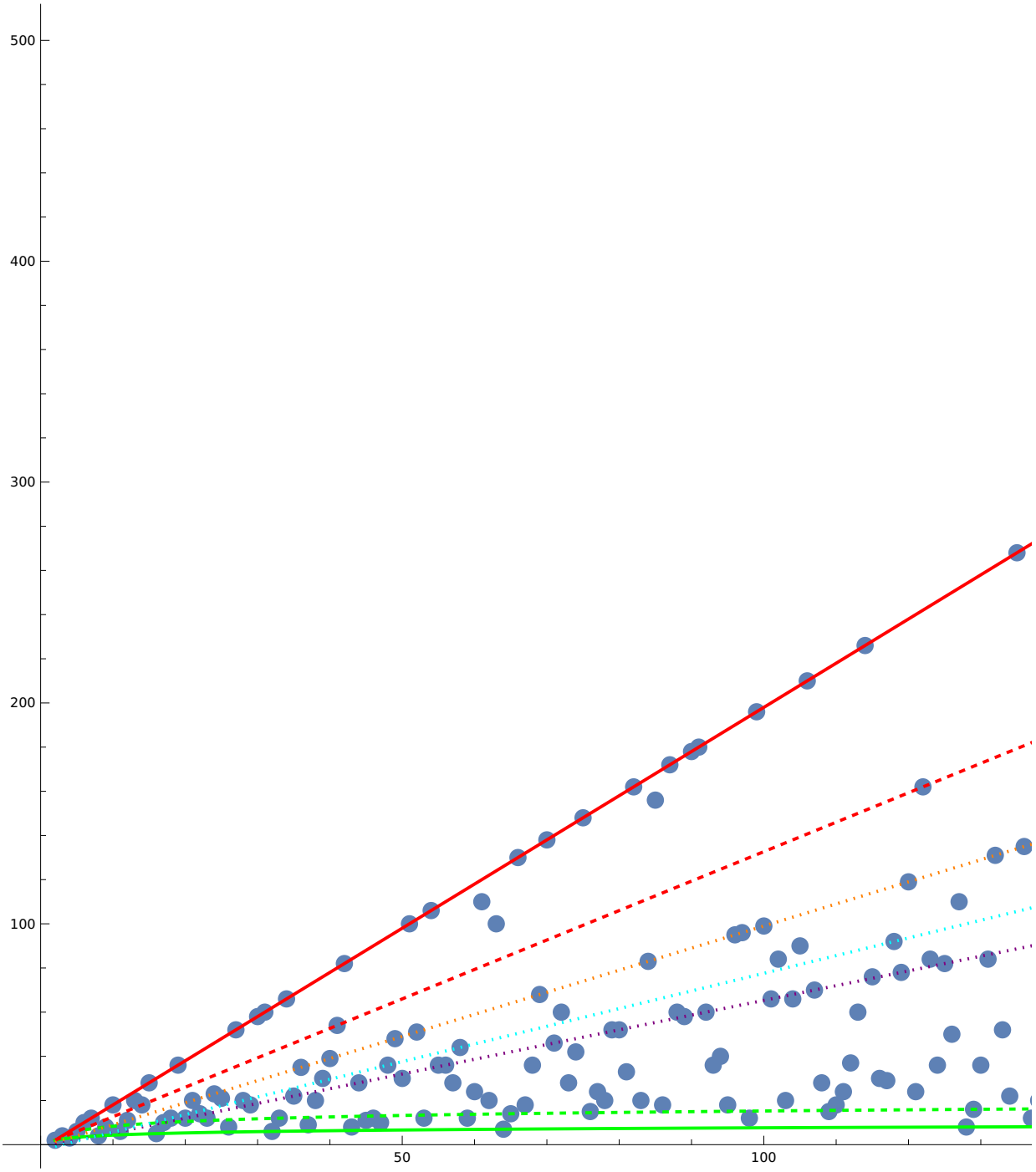
In[29]:=
```
yyyPlot = Plot[4 / 5 x - 12 / 5, {x, 2, NUM}, PlotStyle → {Cyan, Dotted}]
```

Out[29]=



In[30]:=
```
Show[shuffleData, upperBound, lowerBound,
 lower2Plot, subUpper, halfUpper, xxxPlot, yyyPlot]
```

Out[30]=



In[31]:=
```
NUM1 = 250; NUM2 = 300;
Results2 = {};
For[k = NUM1, k ≤ NUM2, k++,
cards = Range[2 * k]; (* create list {1, 2, 3, ...} *)
cards0 = cards; (* make copy of original list *)
ShuffleCount = 0;
cards2 = {};
While[cards0 ≠ cards2,
cards2 = {};
(* create shuffled list*)
For[i = 1, i ≤ Length[cards], i++,
If[Mod[i, 2] == 1,
AppendTo[cards2, cards[[(i + 1) / 2]]],
AppendTo[cards2, cards[[Length[cards] / 2 + i / 2]]]
](*end if*)
](*end for loop over cards*);
ShuffleCount++;
(*Print["Count: ",ShuffleCount, " List: ",cards2];*)
cards = cards2;
];
MyTuple = {};
AppendTo[MyTuple, k];
AppendTo[MyTuple, ShuffleCount];
AppendTo[Results2, MyTuple];
](*  k loop *); Print[Results2];
```

```
{{250, 166}, {251, 166}, {252, 251}, {253, 100}, {254, 156}, {255, 508}, {256, 9},
 {257, 18}, {258, 204}, {259, 230}, {260, 172}, {261, 260}, {262, 522}, {263, 60},
 {264, 40}, {265, 253}, {266, 174}, {267, 60}, {268, 212}, {269, 178}, {270, 210},
 {271, 540}, {272, 180}, {273, 36}, {274, 546}, {275, 60}, {276, 252}, {277, 39},
 {278, 36}, {279, 556}, {280, 84}, {281, 40}, {282, 562}, {283, 28}, {284, 54}, {285, 284},
 {286, 114}, {287, 190}, {288, 220}, {289, 144}, {290, 96}, {291, 246}, {292, 260},
 {293, 12}, {294, 586}, {295, 90}, {296, 196}, {297, 148}, {298, 24}, {299, 198}, {300, 299}}
```

In[37]:=
```
NUM1 = 300; NUM2 = 350;
Results3 = {};
For[k = NUM1, k ≤ NUM2, k++,
cards = Range[2 * k]; (* create list {1, 2, 3, ...} *)
cards0 = cards; (* make copy of original list *)
ShuffleCount = 0;
cards2 = {};
While[cards0 ≠ cards2,
cards2 = {};
(* create shuffled list*)
For[i = 1, i ≤ Length[cards], i++,
If[Mod[i, 2] == 1,
AppendTo[cards2, cards[[(i + 1) / 2]]],
AppendTo[cards2, cards[[Length[cards] / 2 + i / 2]]]
](*end if*)
](*end for loop over cards*);
ShuffleCount++;
(*Print["Count: ",ShuffleCount, " List: ",cards2];*)
cards = cards2;
];
MyTuple = {};
AppendTo[MyTuple, k];
AppendTo[MyTuple, ShuffleCount];
AppendTo[Results3, MyTuple];
](*  k loop *); Print[Results3];
```

```
{{300, 299}, {301, 25}, {302, 66}, {303, 220}, {304, 303}, {305, 84}, {306, 276},
 {307, 612}, {308, 20}, {309, 154}, {310, 618}, {311, 198}, {312, 33}, {313, 500},
 {314, 90}, {315, 72}, {316, 45}, {317, 210}, {318, 28}, {319, 84}, {320, 210}, {321, 64},
 {322, 214}, {323, 28}, {324, 323}, {325, 290}, {326, 30}, {327, 652}, {328, 260},
 {329, 18}, {330, 658}, {331, 660}, {332, 24}, {333, 36}, {334, 308}, {335, 74},
 {336, 60}, {337, 48}, {338, 180}, {339, 676}, {340, 48}, {341, 226}, {342, 22},
 {343, 68}, {344, 76}, {345, 156}, {346, 230}, {347, 30}, {348, 276}, {349, 40}, {350, 58}}
```