

Using Wolfram-Cloud/Mathematica to explore the Perfect Shuffle Problem

<https://www.youtube.com/watch?v=tBGZ7cTVHFo>

<http://www1.lasalle.edu/~blum/c152wks/PerfectShuffle.pdf>

Riffle

```
In[1]:= cards = Range[20]; (* create list {1, 2, 3, ...} -- deck of cards *)
cards0 = cards; (* make copy of original list *)
ShuffleCount = 0;
cards2 = {};
While[cards0 != cards2,
  cards2 = {};
  (* create shuffled list*)
  For[i = 1, i ≤ Length[cards], i++,
    If[Mod[i, 2] == 1,
      AppendTo[cards2, cards[(i + 1)/2]],
      AppendTo[cards2, cards[Length[cards]/2 + i/2]]
    ](*end if*)
  ](*end for loop over cards*);
  ShuffleCount++;
  Print["Count: ", ShuffleCount,
    " List: ", cards2, " 2 location: ", Position[cards2, 2]];
  cards = cards2;
]
```

Count: 1 List:

{1, 11, 2, 12, 3, 13, 4, 14, 5, 15, 6, 16, 7, 17, 8, 18, 9, 19, 10, 20} 2 location: {{3}}

Count: 2 List:

{1, 6, 11, 16, 2, 7, 12, 17, 3, 8, 13, 18, 4, 9, 14, 19, 5, 10, 15, 20} 2 location: {{5}}

Count: 3 List:

{1, 13, 6, 18, 11, 4, 16, 9, 2, 14, 7, 19, 12, 5, 17, 10, 3, 15, 8, 20} 2 location: {{9}}

Count: 4 List:

{1, 7, 13, 19, 6, 12, 18, 5, 11, 17, 4, 10, 16, 3, 9, 15, 2, 8, 14, 20} 2 location: {{17}}

Count: 5 List:

{1, 4, 7, 10, 13, 16, 19, 3, 6, 9, 12, 15, 18, 2, 5, 8, 11, 14, 17, 20} 2 location: {{14}}

Count: 6 List:

{1, 12, 4, 15, 7, 18, 10, 2, 13, 5, 16, 8, 19, 11, 3, 14, 6, 17, 9, 20} 2 location: {{8}}

Count: 7 List:

{1, 16, 12, 8, 4, 19, 15, 11, 7, 3, 18, 14, 10, 6, 2, 17, 13, 9, 5, 20} 2 location: {{15}}

Count: 8 List:

{1, 18, 16, 14, 12, 10, 8, 6, 4, 2, 19, 17, 15, 13, 11, 9, 7, 5, 3, 20} 2 location: {{10}}

Count: 9 List:

{1, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 20} 2 location: {{19}}

Count: 10 List:

{1, 10, 19, 9, 18, 8, 17, 7, 16, 6, 15, 5, 14, 4, 13, 3, 12, 2, 11, 20} 2 location: {{18}}

Count: 11 List:

{1, 15, 10, 5, 19, 14, 9, 4, 18, 13, 8, 3, 17, 12, 7, 2, 16, 11, 6, 20} 2 location: {{16}}

Count: 12 List:

{1, 8, 15, 3, 10, 17, 5, 12, 19, 7, 14, 2, 9, 16, 4, 11, 18, 6, 13, 20} 2 location: {{12}}

Count: 13 List:

{1, 14, 8, 2, 15, 9, 3, 16, 10, 4, 17, 11, 5, 18, 12, 6, 19, 13, 7, 20} 2 location: {{4}}

Count: 14 List:

{1, 17, 14, 11, 8, 5, 2, 18, 15, 12, 9, 6, 3, 19, 16, 13, 10, 7, 4, 20} 2 location: {{7}}

Count: 15 List:

{1, 9, 17, 6, 14, 3, 11, 19, 8, 16, 5, 13, 2, 10, 18, 7, 15, 4, 12, 20} 2 location: {{13}}

Count: 16 List:

{1, 5, 9, 13, 17, 2, 6, 10, 14, 18, 3, 7, 11, 15, 19, 4, 8, 12, 16, 20} 2 location: {{6}}

Count: 17 List:

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20} 2 location: {{11}}

Count: 18 List:

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20} 2 location: {{2}}

```
In[6]:= NUM = 10;  
Pos = 2;  
SCount = 0;  
For[i = 0, i < 2 * NUM - 2, i++,  
SCount++;  
Pos = Mod[Pos + 2 ^ i, 2 * NUM - 1];  
Print[Pos];  
If[Pos == 2, Break[],];  
]; Print[SCount];  
3  
5  
9  
17  
14  
8  
15  
10  
0  
18  
16  
12  
4  
7  
13  
6  
11  
2  
18
```

```

In[10]:= Results = {};
For[j = 1, j < 1000, j++,
Pos = 2;
SCount = 0;
For[i = 0, i < 2*j - 2, i++,
SCount++;
Pos = Mod[Pos + 2^i, 2*j - 1];
(*Print[Pos]; *)
If[Pos == 2, Break[],]; (* Using Break instead of using While *)
]; (*Print[SCount]; *)
MyTuple = {};
AppendTo[MyTuple, j];
AppendTo[MyTuple, SCount];
AppendTo[Results, MyTuple];
];
Print[Results];

```

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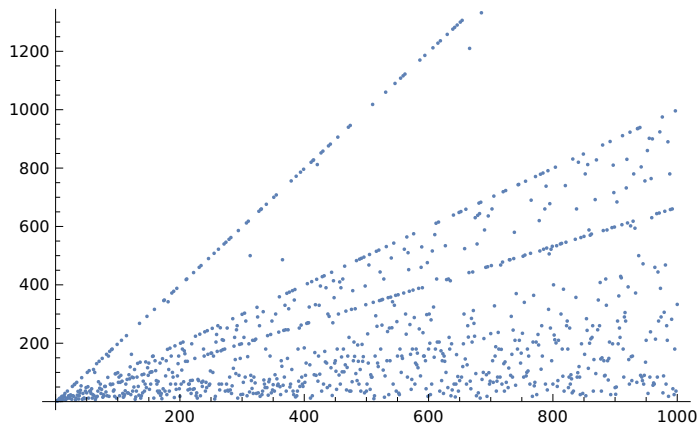
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```

```
In[12]:= ListPlot[Results]
```

```
Out[12]=
```



Pulling out the small results that do not belong to the two previously identified patterns

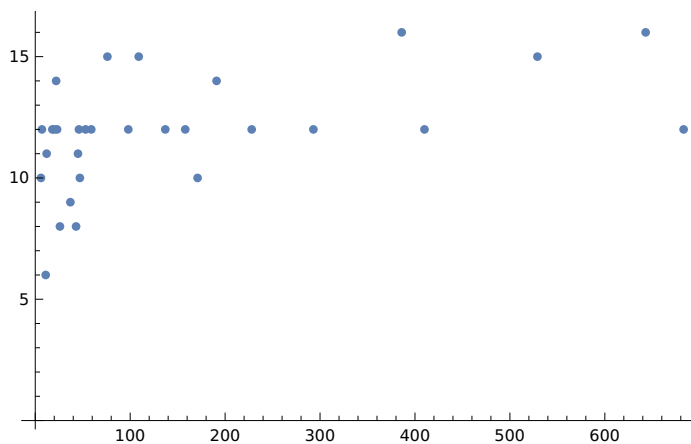
```
In[13]:= smalls = Select[Results, #[[2]] ≤ 16 && Log[2, #[[1]] + 1 ≠ #[[2]] && 2 * (Log[2, #[[1]] - 1] + 1) ≠ #[[2]] &]
```

```
Out[13]=
```

```
{{6, 10}, {7, 12}, {11, 6}, {12, 11}, {18, 12}, {20, 12}, {22, 14}, {23, 12}, {26, 8}, {37, 9}, {43, 8},
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```

```
In[14]:= ListPlot[smalls]
```

```
Out[14]=
```



```
In[15]:= For[i = 1, i ≤ Length[smalls], i++,
Print[BaseForm[smalls[[i]][[1], 2], " ", smalls[[i]][[1], " ", smalls[[i]][[2]]];
]
```

110_2 6 10
 111_2 7 12
 1011_2 11 6
 1100_2 12 11
 10010_2 18 12
 10100_2 20 12
 10110_2 22 14
 10111_2 23 12
 11010_2 26 8
 100101_2 37 9
 101011_2 43 8
 101101_2 45 11
 101110_2 46 12
 101111_2 47 10
 110101_2 53 12
 111011_2 59 12
 1001100_2 76 15
 1100010_2 98 12
 1101101_2 109 15
 10001001_2 137 12
 10011110_2 158 12
 10101011_2 171 10
 10111111_2 191 14
 11100100_2 228 12
 100100101_2 293 12
 110000010_2 386 16
 110011010_2 410 12
 1000010001_2 529 15
 1010000011_2 643 16
 1010101011_2 683 12

Possible Pattern I

$x=2^3+2^1+1=11, y=6$
 $x=2^5+2^3+2^1+1=43, y=8$
 $x=2^7+2^5+2^3+2^1+1=171, y=10$
 $x=2^9+2^7+2^5+2^3+2+1=683, y=12$

Possible Pattern II

$x=2^2+1=5, y=6$

$$x=2^5+2^2+1=37, y=9$$

$$x=2^8+2^5+2^2+1=293, y=12$$

$$x=2^{11}+2^8+2^5+2^2+1=2341, y=15$$

Possible Pattern III

$$x=2^4+2^3+2=26, y=8$$

$$x=2^6+2^5+2=98, y=12$$

$$x=2^8+2^7+2=386, y=16$$

$$x=2^{10}+2^9+2=1538, y=20$$

Possible Pattern IV

$$x=10111 \rightarrow 23, y=12$$

$$x=1011111 \rightarrow 95, y=18$$

$$x=101111111 \rightarrow 383, y=24$$

$$x=10111111111 \rightarrow 1535, y=30$$

Possible Pattern V

$$x=101111 \rightarrow 47, y=10$$

$$x=10111111 \rightarrow 191, y=14$$

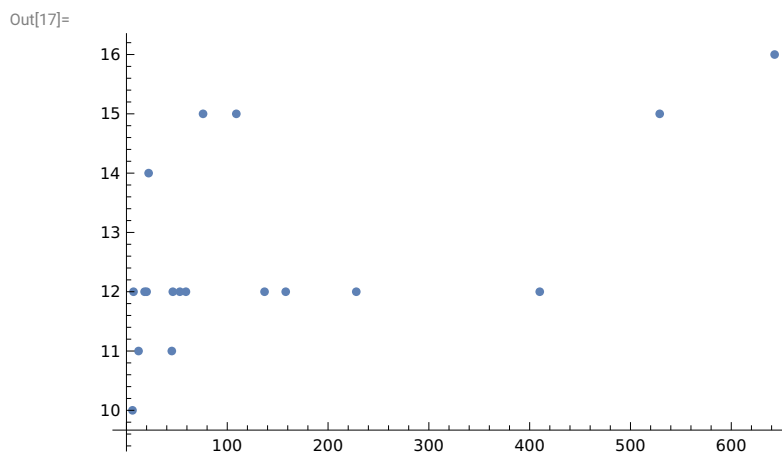
$$x=1011111111 \rightarrow 767, y=18$$

$$x=101111111111 \rightarrow 3071, y=22$$

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In[16]:= left = {{6, 10}, {7, 12}, {12, 11}, {18, 12}, {20, 12}, {22, 14}, {45, 11}, {46, 12}, {53, 12},
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```

```
In[17]:= ListPlot[left]
```



```
In[18]:= For[i = 1, i ≤ Length[left], i++,
  Print[BaseForm[left[[i]][1], 2], " ", left[[i]][1], " ", left[[i]][2]]];
]
```

110₂ 6 10

111₂ 7 12

1100₂ 12 11

10010₂ 18 12

10100₂ 20 12

10110₂ 22 14

101101₂ 45 11

101110₂ 46 12

110101₂ 53 12

111011₂ 59 12

1001100₂ 76 15

1101101₂ 109 15

10001001₂ 137 12

10011110₂ 158 12

11100100₂ 228 12

110011010₂ 410 12

1000010001₂ 529 15

1010000011₂ 643 16

```

In[4]:= Results2 = {};
For[j = 1001, j < 2000, j ++;
Pos = 2;
SCount = 0;
For[i = 0, i < 2*j - 2, i ++,
SCount++;
Pos = Mod[Pos + 2^i, 2*j - 1];
(*Print[Pos]; *)
If[Pos == 2, Break[],];
]; (*Print[SCount]; *)
MyTuple = {};
AppendTo[MyTuple, j];
AppendTo[MyTuple, SCount];
AppendTo[Results2, MyTuple];
];
Print[Results2];
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For[j = 2001, j < 3000, j++,
Pos = 2;
SCount = 0;
For[i = 0, i < 2*j - 2, i++,
SCount++;
Pos = Mod[Pos + 2^i, 2*j - 1];
(*Print[Pos]; *)
If[Pos == 2, Break[],];
]; (*Print[SCount]; *)
MyTuple = {};
AppendTo[MyTuple, j];
AppendTo[MyTuple, SCount];
AppendTo[Results3, MyTuple];
];
Print[Results3];

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