

Perfect-Shuffle (Riffle) sequence -- More patterns within

Using Wolfram-Cloud/Mathematica to explore the Perfect Shuffle Problem

<https://www.youtube.com/watch?v=tBGZ7cTVHFo>

More on the Perfect Shuffle (Riffle) using Wolfram-Cloud/Mathematica

https://www.youtube.com/watch?v=arj_Q3KE7T0

<https://oeis.org/A002326>

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In[30]:= Results = {};
For[j = 1, j < 2000, j++,
  Pos = 2;
  SCount = 0;
  For[i = 0, i < 2*j - 2, i++,
    SCount++;
    Pos = Mod[Pos + 2^i, 2*j - 1];
    (*Print[Pos]; *)
  If[Pos == 2, Break[],] (* Using Break instead of using While *)
]; (*Print[SCount]; *)
MyTuple = {};
AppendTo[MyTuple, j];
AppendTo[MyTuple, SCount];
AppendTo[Results, MyTuple];
];
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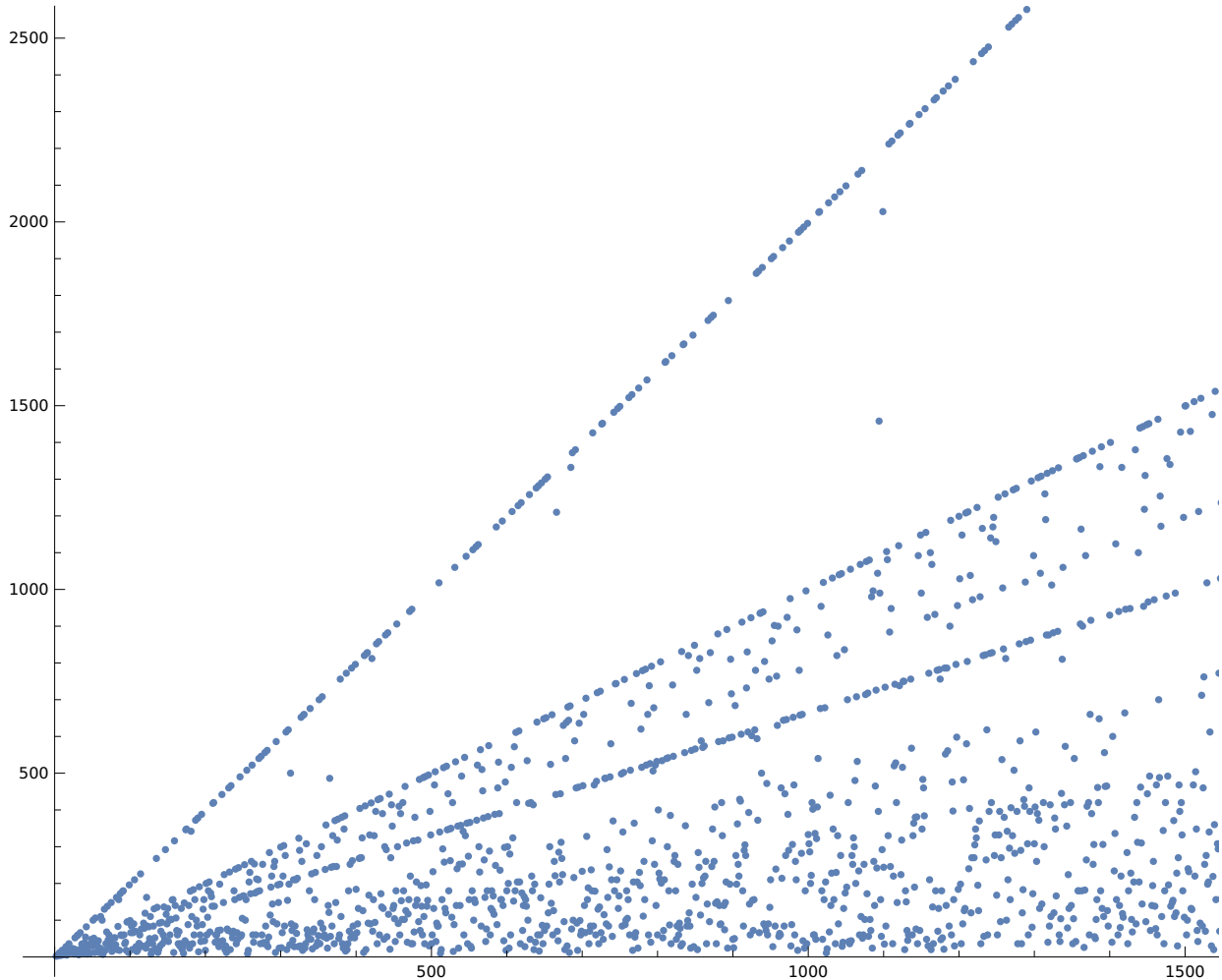
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```

```
In[46]:= ListPlot[Results]
```

Out[46]=



```
In[47]:= subUppers = Select[Results, 4/3*(#[[1]] + 1) - 2 == #[[2]] &]
```

Out[47]=

```
{{2, 2}, {5, 6}, {14, 18}, {41, 54}, {122, 162}, {365, 486}, {1094, 1458}}
```

```
In[48]:= For[i = 1, i ≤ Length[subUppers], i++,  
Print[BaseForm[subUppers[[i]][[1], 3], " ", subUppers[[i]][[1], " ", subUppers[[i]][[2]]];  
]
```

```
23 2 2
```

```
123 5 6
```

```
1123 14 18
```

```
11123 41 54
```

```
111123 122 162
```

```
1111123 365 486
```

```
11111123 1094 1458
```

Pattern A

$x=3^{1-1} \rightarrow 2, y=2=(4x-2)/3$

$x=3^2-3^{1-1} \rightarrow 5, y=6=(4x-2)/3$
 $x=3^3-3^2-3^{1-1} \rightarrow 14, y=18$
 $x=3^4-3^3-3^2-3^{1-1} \rightarrow 41, y=54$

```

In[32]:= tweens = Select[Results, 4/3*(#[[1]] + 1) - 2 < #[[2]] && #[[1]]*2 - 2 > #[[2]] &]
Out[32]=
{{13, 20}, {61, 110}, {63, 100}, {85, 156}, {181, 342}, {313, 500}, {421, 812}, {666, 1210},
 {685, 1332}, {1099, 2028}, {1405, 2756}, {1563, 2500}, {1741, 3422}, {1861, 3660}}

In[33]:= For[i = 1, i ≤ Length[tweens], i++,
  Print[BaseForm[tweens[[i]][[1]], 13], " ", tweens[[i]][[1]],
    " ", tweens[[i]][[2]], " ", N[(tweens[[i]][[2]] + 2)/(tweens[[i]][[1]] + 1)]];
]

1013 13 20 1.57143
4913 61 110 1.80645
4b13 63 100 1.59375
6713 85 156 1.83721
10c13 181 342 1.89011
1b113 313 500 1.59873
26513 421 812 1.92891
3c313 666 1210 1.81709
40913 685 1332 1.94461
66713 1099 2028 1.84545
84113 1405 2756 1.96159
93313 1563 2500 1.59974
a3c13 1741 3422 1.96556
b0213 1861 3660 1.9667

```

Pattern B

Those with slope a bit over 1.5

$x=13 \rightarrow \text{BaseForm}[13,5]=23, y=20$

$x=63 \rightarrow \text{BaseForm}[63,5]=223, y=100$

$x=313 \rightarrow \text{BaseForm}[313,5]=2223, y=500$

Equation: $y=8/5x - 4/5$

Check: $x=1563, y=2500$ fits pattern

Pattern C

Those with slope around 1.8

$x=61 \rightarrow \text{BaseForm}[61,11]=56, y=110$

$x=666 \rightarrow \text{BaseForm}[666,11]=556, y=1210$

Equation: $y=20/11 x - 10/11$

TO CHECK: $x=7321 \rightarrow \text{BaseForm}[7321, 11]=5556$

Pattern D

Those with slope around 1.84

$x=85 \rightarrow \text{BaseForm}[85,13]=67, y=156$

$x=1099 \rightarrow \text{BaseForm}[1099, 13]=667, y=2028$

Equation: $y=24/13 x - 12/13$

TO CHECK: $x=14281 \rightarrow \text{BaseForm}[14281, 13]=6667$

What happened to Base 7?

Pattern E

$x=25 \rightarrow \text{BaseForm}[25,7]=34, y=21$

$x=172 \rightarrow \text{BaseForm}[172,7]=334, y=147$

$x=1029 \rightarrow \text{BaseForm}[1029,7]=3334, y=1029$

Equation: $y=6/7 x - 3/7$ (-- slope less than one --)

TO CHECK: $x=8404 \rightarrow \text{BaseForm}[8404,7]=33334$

What about Base 9? (Nine is not prime)

(Partial) Pattern F

$x=41 \rightarrow \text{BaseForm}[41,9]=45, y=54$

$x=365 \rightarrow \text{BaseForm}[365,9]=445, y=486$

$x=3281 \rightarrow \text{BaseForm}[3281,9]=4445, y=4374$

Equation: $y = 4/3 x - 2/3$

This is just part of Pattern A