Previously looked at "z-score confidence intervals"
For 95% confidence
Ave_sample - 1.96*SD_population/sqrt(sampleNum) < Ave_population < Ave_sample + 1.96*SD_population/sqrt(sampleNum)
To replace SD_population with SD_sample we move to the Student T Distribution

In[1]:=
```
numSamples = 2
```
Out[1]=
```
2
```

In[2]:=
```
v = RandomVariate[NormalDistribution[0, 1], numSamples]
```
Out[2]=
```
{0.0917503, -0.337477}
```

In[3]:=
```
mu = Mean[v]
```
Out[3]=
```
-0.122863
```

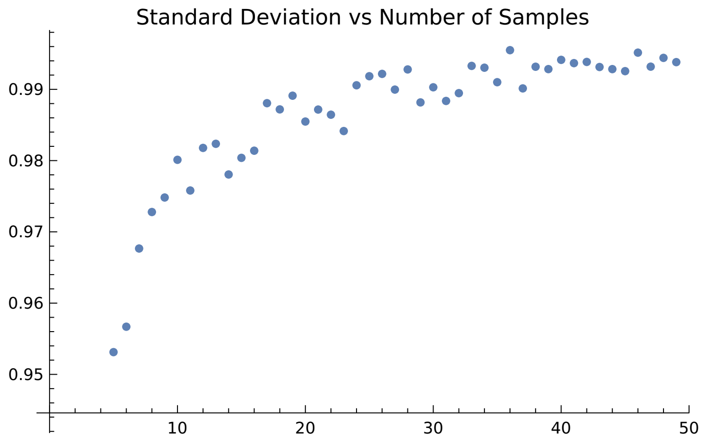
In[4]:=
```
sd = StandardDeviation[v]
```
Out[4]=
```
0.303509
```

In[5]:=
```
mySD = {}; For[j = 2, j ≤ 50, j++,
myList = {}; For[i = 1, i ≤ 5000, i++,
v = RandomVariate[NormalDistribution[0, 1], j];
sd = StandardDeviation[v];
AppendTo[myList, sd]
]; AppendTo[mySD, Mean[myList]]
];
Print[mySD]
```

```
{0.808746, 0.890562, 0.91953, 0.939481, 0.953122, 0.956688, 0.967662, 0.972784, 0.974819,
 0.98011, 0.975802, 0.98178, 0.982358, 0.978051, 0.980386, 0.981389, 0.988057,
 0.987185, 0.989107, 0.98548, 0.987162, 0.98644, 0.984147, 0.99057, 0.991841,
 0.992169, 0.989962, 0.992792, 0.988165, 0.990288, 0.98837, 0.989466, 0.993289,
 0.993037, 0.991002, 0.995496, 0.990131, 0.993172, 0.992839, 0.994136, 0.993676,
 0.993847, 0.993134, 0.992837, 0.992552, 0.995149, 0.993182, 0.994415, 0.993818}
```

In[6]:=
```
ListPlot[mySD, PlotLabel → "Standard Deviation vs Number of Samples",
 PlotLabel → {"Number of Samples", "Standard Deviation"}]
```

Out[6]=

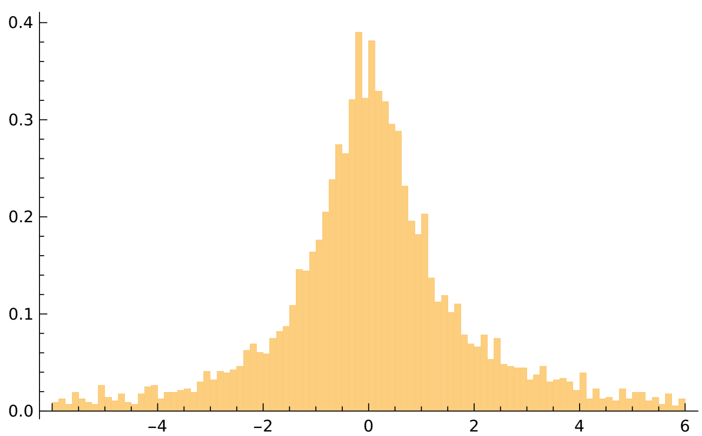


In[7]:=
```
numSamples = 2; myList = {}; For[i = 1, i ≤ 5000, i++,
v = RandomVariate[NormalDistribution[0, 1], numSamples];
mu = Mean[v]; sd = StandardDeviation[v];
AppendTo[myList, mu / (sd / Sqrt[numSamples])]
]; Print["done"]
```

done

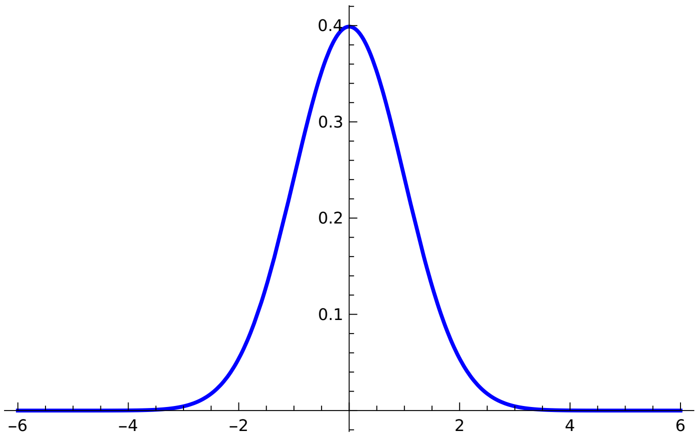
In[8]:=
```
h1 = Histogram[myList, {-6, 6, 0.125}, "PDF"]
```

Out[8]=



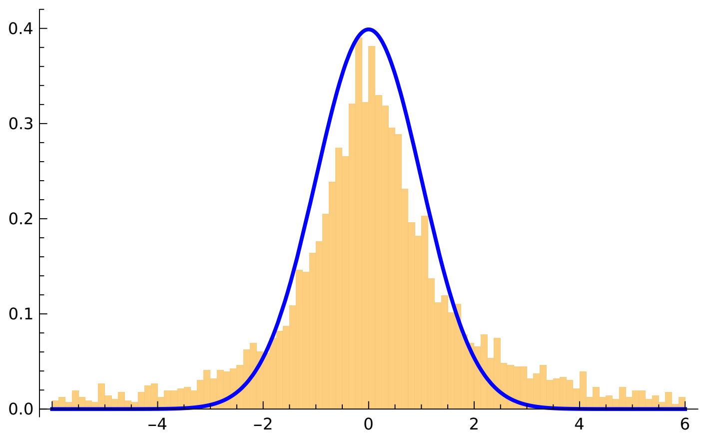
In[9]:= `h2 = Plot[PDF[NormalDistribution[0, 1], x], {x, -6, 6}, PlotStyle → {Blue}]`

Out[9]=



In[10]:= `Show[h1, h2]`

Out[10]=



In[11]:= `listSD = StandardDeviation[myList]`

Out[11]=

166.949

In[12]:= `PDF[StudentTDistribution[1], x]`

Out[12]=

$$\frac{1}{\pi (1 + x^2)}$$

In[13]:= `PDF[StudentTDistribution[2], x]`

Out[13]=

$$\left(\frac{1}{2 + x^2}\right)^{3/2}$$

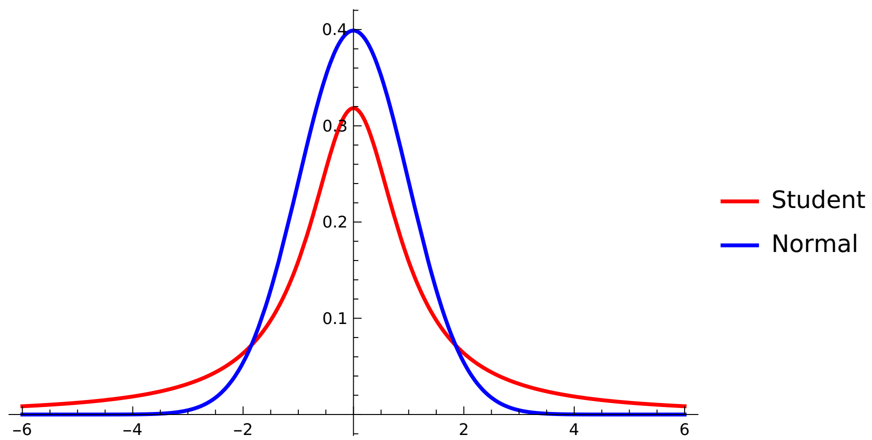
In[14]:= `PDF[StudentTDistribution[3], x]`

Out[14]=

$$\frac{6\sqrt{3}}{\pi (3 + x^2)^2}$$

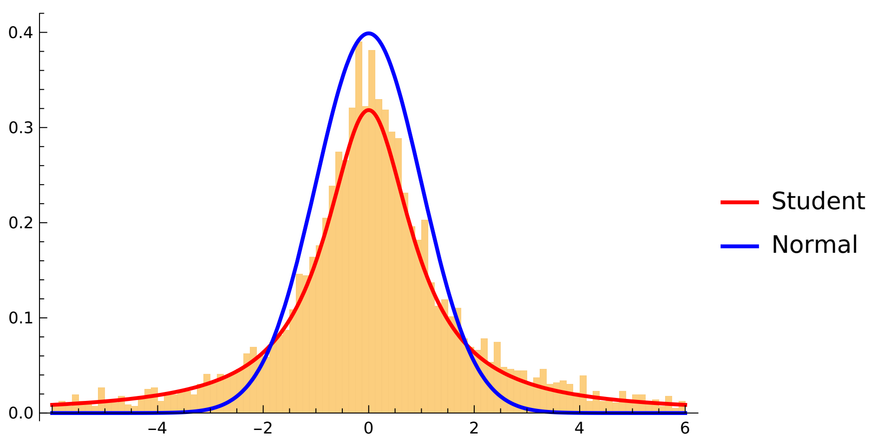
In[15]:=
```
h3 = Plot[{PDF[StudentTDistribution[numSamples - 1], x],
PDF[NormalDistribution[0, 1], x]}, {x, -6, 6},
PlotStyle → {Red, Blue}, PlotLegends → {"Student", "Normal"}]
```

Out[15]=



In[16]:=
```
Show[h1, h3]
```

Out[16]=



Remember when looking up the "z" value in the Student T Test table -- that the degrees of freedom is one less than the sample number.   Mistake 1. conf=1.96 Mistake 2. conf=3.182 Correct for sampleNum=3 (degree of freedom = 2) conf=4.303

In[25]:=
```
AmIn = 0; sampleNum = 3; conf = 4.303; result = {}; For[i = 1, i ≤ 1000, i++,
mu = Random[Real, {0, 10}]; sig = Random[Real, {1, 5}];
samples = RandomVariate[NormalDistribution[mu, sig], sampleNum];
sampleAve = Mean[samples]; sampleSD = StandardDeviation[samples];
DataPoint = {}; AppendTo[DataPoint, mu]; AppendTo[DataPoint, sig];
AppendTo[DataPoint, ave];
within = If[mu > sampleAve + sampleSD * conf / Sqrt[sampleNum], 0,
If[mu < sampleAve - sampleSD * conf / Sqrt[sampleNum], 0, 1]];
AmIn = AmIn + within;
AppendTo[DataPoint, within];
 AppendTo[result, DataPoint];
]; Print[AmIn]
```

```
951
```