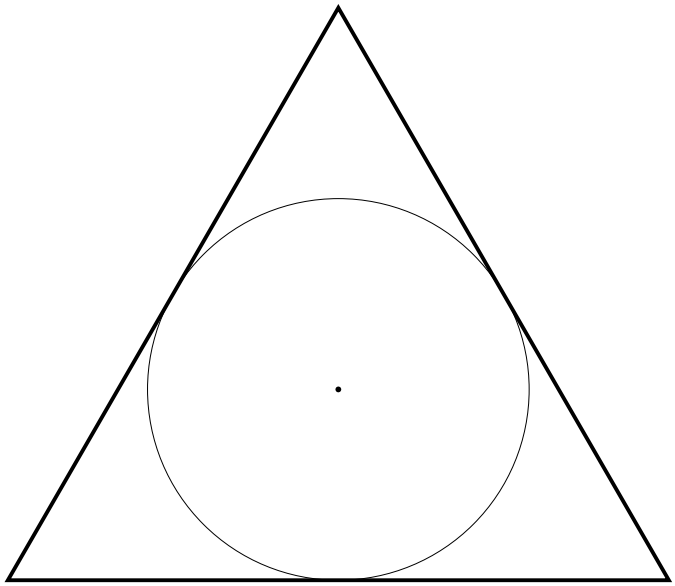
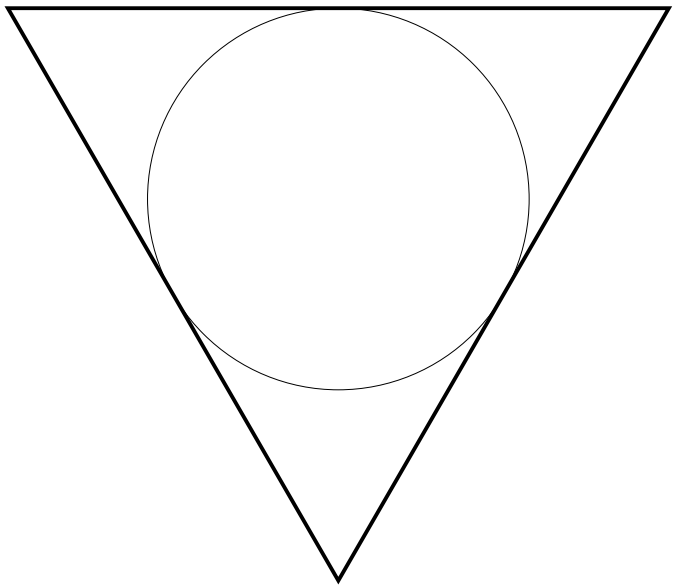
In[1]:=
```
Show[Graphics[Circle[{0, 0}, 1]],
 Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2, 3]}], Graphics[Point[{0, 0}]]]
```

Out[1]=



In[2]:=
```
Show[Graphics[Circle[{0, 0}, 1]],
 Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, {2, Pi / 6}, 3]}]]
```

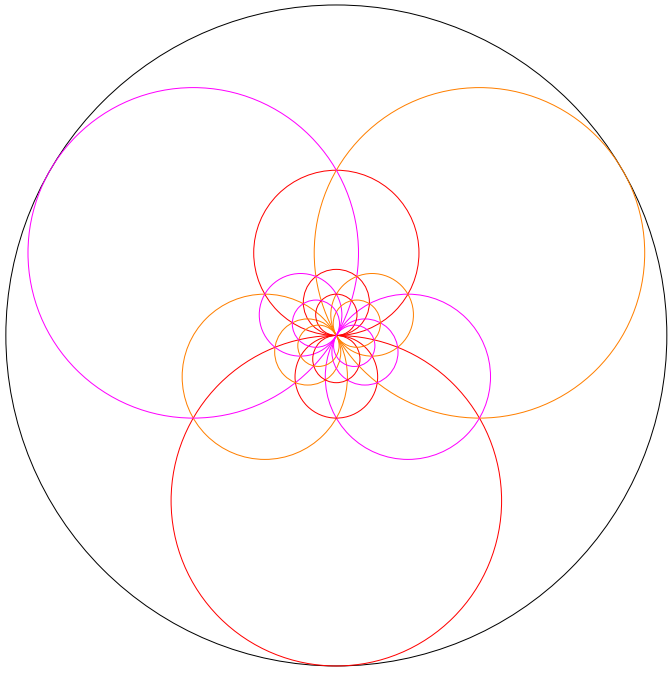
Out[2]=



Circle inversion of the lines of the triangular lattice lead to six sets of circle -- with every circle passing through the origin of the "circle of inversion"

In[3]:=
```
TLI = {};
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TLI, center];
For[i = 0, i < 3, i++,
(* horizontal lines above *)
HA = Graphics[{Red, Circle[{0, 1/(4 + 6*i)}, 1/(4 + 6*i)]}];
AppendTo[TLI, HA];
(* horizontal lines below *)
HB = Graphics[{Red, Circle[{0, -1/(2 + 6*i)}, 1/(2 + 6*i)]}];
AppendTo[TLI, HB];
(* negative slope to right *)
NSR =
  Graphics[{Orange, Circle[{1/(2 + 6*i)*Cos[Pi/6], 1/(2 + 6*i)*Sin[Pi/6]}, 1/(2 + 6*i)]}];
AppendTo[TLI, NSR];
(* negative slope to left *)
NSR = Graphics[
   {Orange, Circle[{1/(4 + 6*i)*Cos[Pi + Pi/6], 1/(4 + 6*i)*Sin[Pi + Pi/6]}, 1/(4 + 6*i)]}];
AppendTo[TLI, NSR];
(* positive slope to right *)
NSR =
  Graphics[{Magenta, Circle[{1/(4 + 6*i)*Cos[-Pi/6], 1/(4 + 6*i)*Sin[-Pi/6]}, 1/(4 + 6*i)]}];
AppendTo[TLI, NSR];
(* Positive slope to left *)
NSR = Graphics[
   {Magenta, Circle[{1/(2 + 6*i)*Cos[Pi - Pi/6], 1/(2 + 6*i)*Sin[Pi - Pi/6]}, 1/(2 + 6*i)]}];
AppendTo[TLI, NSR];
]; Show[TLI]
```

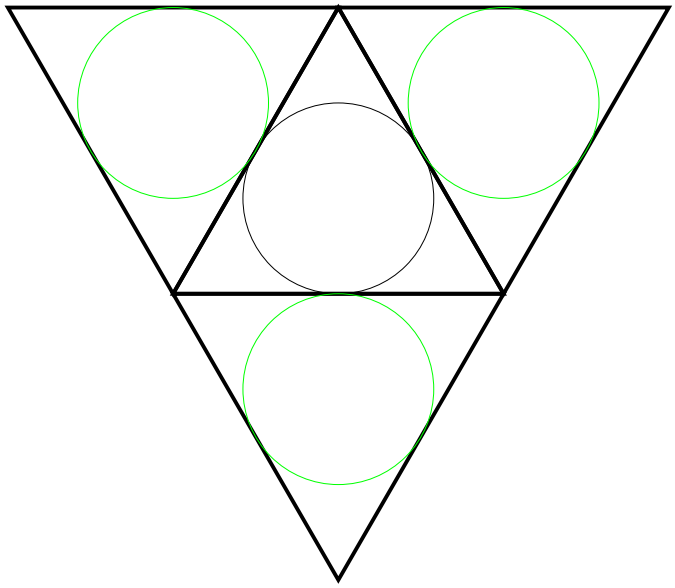
Out[6]=



The "interior regions" are formed by three arcs -- one red, one orange, one magenta

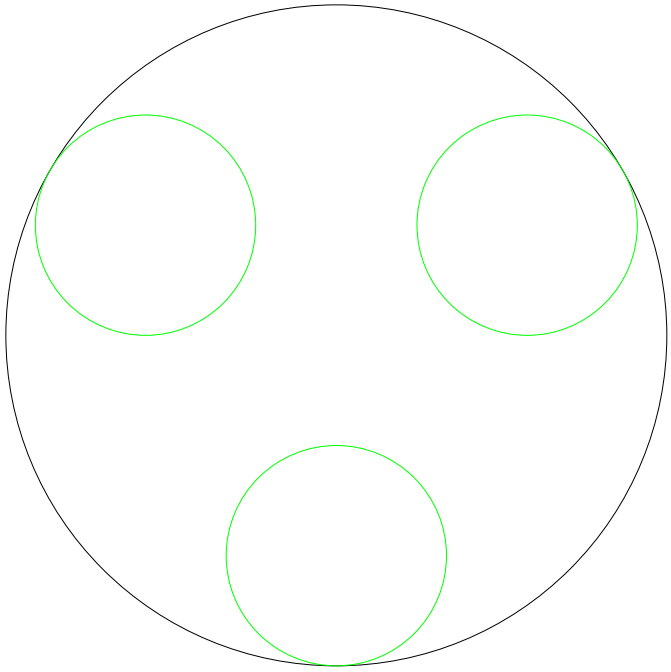
In[50]:=
```
TL = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2, 3]}];
AppendTo[TL, center];
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TL, center];
For[i = 0, i < 3, i++,
NN = Graphics[{EdgeForm[Thick], Opacity[0],
     RegularPolygon[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, {2, Pi / 6}, 3]}];
AppendTo[TL, NN];
NN = Graphics[{Green, Circle[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, 1]}];
AppendTo[TL, NN];
]; Show[TL]
```

Out[55]=



In[13]:=
```
TLI = {};
(* center=Graphics[{EdgeForm[Thick], Opacity[0],RegularPolygon[{0,0},2,3]}];
AppendTo[TLI,center]; *)
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TLI, center];
For[i = 0, i < 3, i++,
NN = Graphics[{Green, Circle[{2/3*Cos[Pi/6+2*Pi/3*i], 2/3*Sin[Pi/6+2*Pi/3*i]}, 1/3]}];
AppendTo[TLI, NN];
]; Show[TLI]
```

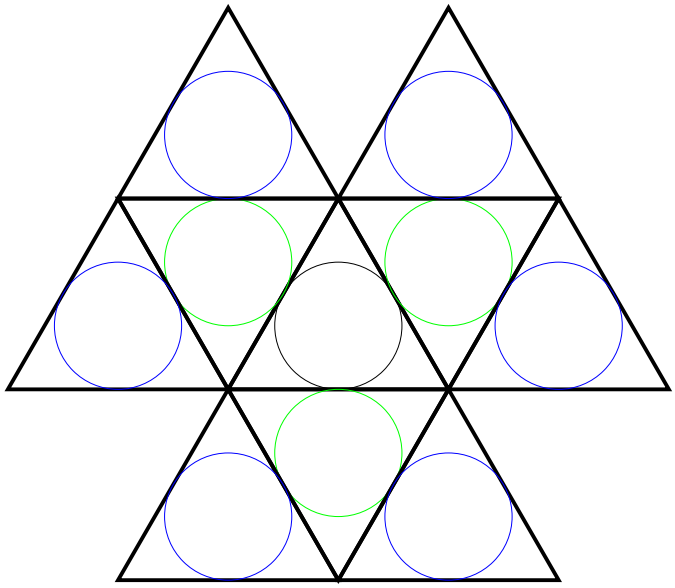
Out[16]=



Previously looked at Circle Inversion of circles inscribed in a honeycomb lattice. The honeycomb lattice and its inversion are more tightly "packed" than what we see above.

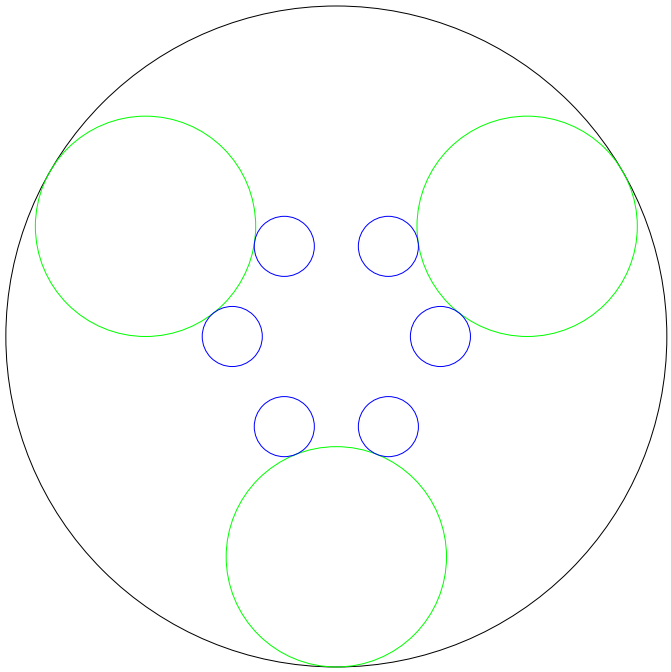
In[17]:=
```
TL = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2, 3]}];
AppendTo[TL, center];
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TL, center];
For[i = 0, i < 3, i++,
NN = Graphics[{EdgeForm[Thick], Opacity[0],
      RegularPolygon[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, {2, Pi / 6}, 3]}];
AppendTo[TL, NN];
NN = Graphics[{Green, Circle[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, 1]}];
AppendTo[TL, NN];
];
For[i = 0, i < 6, i++,
R = 4 * Cos[Pi / 6];
NNN = Graphics[
   {EdgeForm[Thick], Opacity[0], RegularPolygon[{R * Cos[Pi / 3 * i], R * Sin[Pi / 3 * i]}, 2, 3]}];
AppendTo[TL, NNN];
NNN = Graphics[{Blue, Circle[{R * Cos[Pi / 3 * i], R * Sin[Pi / 3 * i]}, 1]}];
AppendTo[TL, NNN];
]; Show[TL]
```

Out[23]=



In[24]:=
```
TLI = {};
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TLI, center];
For[i = 0, i < 3, i++,
NN = Graphics[{Green, Circle[{2/3*Cos[Pi/6 + 2*Pi/3*i], 2/3*Sin[Pi/6 + 2*Pi/3*i]}, 1/3]}];
AppendTo[TLI, NN];
];
For[i = 0, i < 6, i++,
R = 4*Cos[Pi/6]/11;
NNN = Graphics[{Blue, Circle[{R*Cos[Pi/3*i], R*Sin[Pi/3*i]}, 1/11]}];
AppendTo[TLI, NNN];
]; Show[TLI]
```

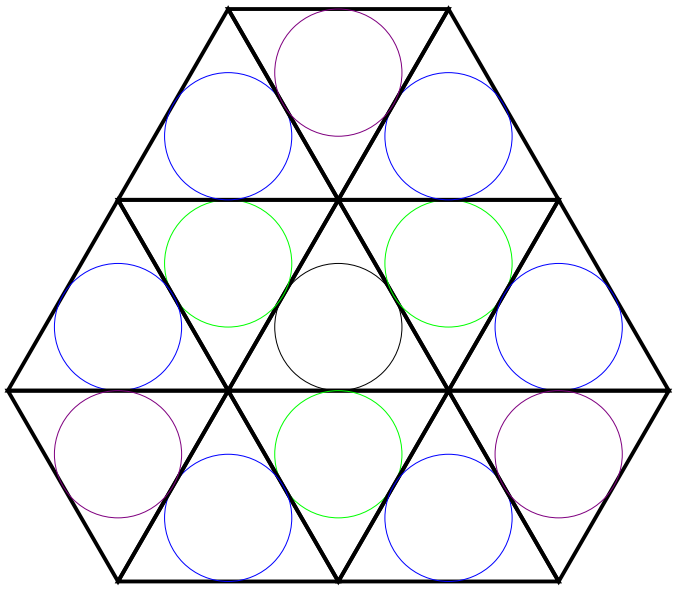
Out[28]=



As in the arrangement that was inverted, each green touches two blues. None of the blues touch each other

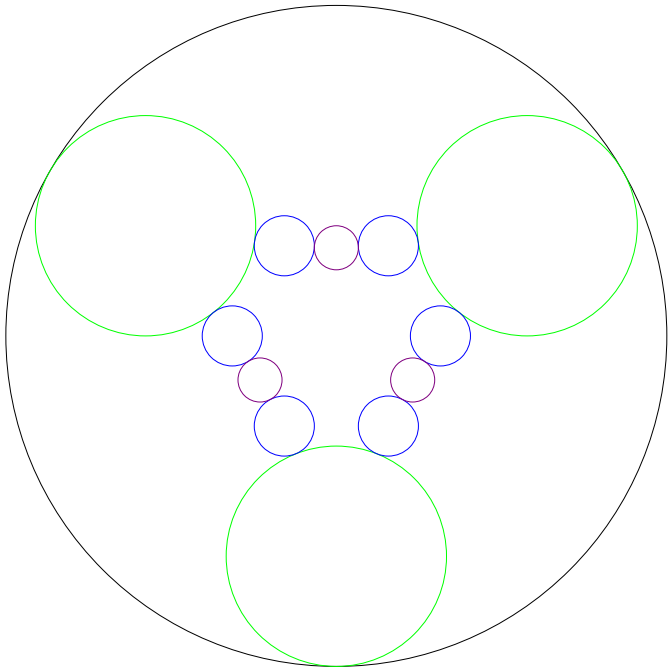
In[29]:=
```
TL = {};
center = Graphics[{EdgeForm[Thick], Opacity[0], RegularPolygon[{0, 0}, 2, 3]}];
AppendTo[TL, center];
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TL, center];
For[i = 0, i < 3, i++,
NN = Graphics[{EdgeForm[Thick], Opacity[0],
    RegularPolygon[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, {2, Pi / 6}, 3]}];
AppendTo[TL, NN];
NN = Graphics[{Green, Circle[{2 * Cos[Pi / 6 + 2 * Pi / 3 * i], 2 * Sin[Pi / 6 + 2 * Pi / 3 * i]}, 1]}];
AppendTo[TL, NN];
];
For[i = 0, i < 6, i++,
R = 4 * Cos[Pi / 6];
NNN = Graphics[
   {EdgeForm[Thick], Opacity[0], RegularPolygon[{R * Cos[Pi / 3 * i], R * Sin[Pi / 3 * i]}, 2, 3]}];
AppendTo[TL, NNN];
NNN = Graphics[{Blue, Circle[{R * Cos[Pi / 3 * i], R * Sin[Pi / 3 * i]}, 1]}];
AppendTo[TL, NNN];
];
For[i = 0, i < 3, i++,
NNNN = Graphics[{EdgeForm[Thick], Opacity[0],
   RegularPolygon[{4 * Cos[Pi / 2 + 2 * Pi / 3 * i], 4 * Sin[Pi / 2 + 2 * Pi / 3 * i]}, {2, Pi / 6}, 3]}];
AppendTo[TL, NNNN];
NNNN = Graphics[{Purple, Circle[{4 * Cos[Pi / 2 + 2 * Pi / 3 * i], 4 * Sin[Pi / 2 + 2 * Pi / 3 * i]}, 1]}];
AppendTo[TL, NNNN];
];
Show[TL]
```

Out[36]=



```
TLI = {};
center = Graphics[Circle[{0, 0}, 1]];
AppendTo[TLI, center];
(* nearest neighbor -- radii 1/3 *)
For[i = 0, i < 3, i++,
NN = Graphics[{Green, Circle[{2/3*Cos[Pi/6 + 2*Pi/3*i], 2/3*Sin[Pi/6 + 2*Pi/3*i]}, 1/3]}];
AppendTo[TLI, NN];
];
(* next-nearest neighbor -- radii 1/11 *)
For[i = 0, i < 6, i++,
R = 4*Cos[Pi/6]/11;
NNN = Graphics[{Blue, Circle[{R*Cos[Pi/3*i], R*Sin[Pi/3*i]}, 1/11]}];
AppendTo[TLI, NNN];
];
(* next-next-nearest neighbor -- radii 1/15 *)
For[i = 0, i < 3, i++,
NNNN =
  Graphics[{Purple, Circle[{4/15*Cos[Pi/2 + 2*Pi/3*i], 4/15*Sin[Pi/2 + 2*Pi/3*i]}, 1/15]}];
AppendTo[TLI, NNNN];
];
Show[TLI]
```

Out[42]=



Put together the inverted lattice edges and the inverted inscribed circles.

In[43]:=

```
TLI = {};
center = Graphics[Circle[{0, 0}, 1]];
```

```
AppendTo[TLI, center];
For[i = 0, i < 3, i++,
NN = Graphics[{Green, Circle[{2/3*Cos[Pi/6 + 2*Pi/3*i], 2/3*Sin[Pi/6 + 2*Pi/3*i]}, 1/3]}];
AppendTo[TLI, NN];
];
For[i = 0, i < 6, i++,
R = 4*Cos[Pi/6]/11;
NNN = Graphics[{Blue, Circle[{R*Cos[Pi/3*i], R*Sin[Pi/3*i]}, 1/11]}];
AppendTo[TLI, NNN];
];
For[i = 0, i < 3, i++,
NNNN = Graphics[
    {Purple, Circle[{4/15*Cos[Pi/2 + 2*Pi/3*i], 4/15*Sin[Pi/2 + 2*Pi/3*i]}, 1/15]}];
AppendTo[TLI, NNNN];
];
For[i = 0, i < 3, i++,
(* horizontal lines above *)
HA = Graphics[{Red, Circle[{0, 1/(4 + 6*i)}, 1/(4 + 6*i)]}];
AppendTo[TLI, HA];
(* horizontal lines below *)
HB = Graphics[{Red, Circle[{0, -1/(2 + 6*i)}, 1/(2 + 6*i)]}];
AppendTo[TLI, HB];
(* negative slope to right *)
NSR =
  Graphics[{Orange, Circle[{1/(2 + 6*i)*Cos[Pi/6], 1/(2 + 6*i)*Sin[Pi/6]}, 1/(2 + 6*i)]}];
AppendTo[TLI, NSR];
(* negative slope to left *)
NSR = Graphics[
   {Orange, Circle[{1/(4 + 6*i)*Cos[Pi + Pi/6], 1/(4 + 6*i)*Sin[Pi + Pi/6]}, 1/(4 + 6*i)]}];
AppendTo[TLI, NSR];
(* positive slope to right *)
NSR =
  Graphics[{Magenta, Circle[{1/(4 + 6*i)*Cos[-Pi/6], 1/(4 + 6*i)*Sin[-Pi/6]}, 1/(4 + 6*i)]}];
AppendTo[TLI, NSR];
(* Positive slope to left *)
NSR = Graphics[
   {Magenta, Circle[{1/(2 + 6*i)*Cos[Pi - Pi/6], 1/(2 + 6*i)*Sin[Pi - Pi/6]}, 1/(2 + 6*i)]}];
AppendTo[TLI, NSR];
];
Show[TLI]
```

Out[49]=