

Classroom Chaos 2 Dr. Richard A. DiDio La Salle University

I. Complex Iteration

By hand (and a calculator if necessary) determine the first four terms of the sequence

$$z_{n+1} = z_n^2 + C$$

where C is a complex constant, and $z_0 = 0 + 0i = (0, 0)$

For C = 0.24+0.1 i, = (0.24, 0.1)

n	Zn			
0	(0,0)			
1				
2				
3				

For C = 0.24 - 0.7 *i* ,= (0.24 , -0.7)

n	Zn			
0	(0,0)			
1				
2				
3				

II. Complex Iteration II: Spreadsheet Day

After doing enough of these things by hand, You have undoubtedly determined by now that, if z = a + bi, then $z^2 = (a^2 - b^2) + 2abiz^2$. Therefore, you can set up this iteration quite easily in a spreadsheet. Here's how:

	А	В	С	D	E
1				.24	.1
2					
3	n	Real z	lmag z		Magnitude
4	0	0	0		
5	1	=B4^2 - C4^2 + \$D\$1	=2*B4*C4 + \$E\$1		=B104^2+C104^2
6	2	=B5^2 - C5^2 + \$D\$1	=2*B5*C5+ \$E\$1		

Some explanation:

Cells B4 and C4 contain z_0 , namely 0 + 0 i, or (0,0)Cells D1 and E1 contain the real and imaginary parts of the arbitrary constant C

Set up this spreadsheet now. Note that copying cells B5 and C5 downward will extend the iteration sequence as far as you'd like within the limitations of your spreadsheet. For now, copy these cells down through B104 and C104.

Also, add the formula in cell E5 (labeled 'Magnitude'). This cell now contains the squared magnitude of the 100th iteration.

Once your spreadsheet is ready, check your values from part 1.

3. Looking for "nice" points in the complex plane

A "nice" point in the complex plane will be defined as a value of the complex parameter C that doesn't cause the iteration to "blow up". "Blowing up" is defined to be a **Magnitude** greater than 4.

To find "nice" points, simply enter a complex number in the "C" region of your sheet (cells D1 and E1), and take a look at the number in cell E5. If this number is 4.0 or greater, your C value is not "nice".

For example, try

C = the nice (.24,0) and C = the not nice (.26,0)

and

C =the nice(-.375,.59) and C =the not nice (-.375,.6)

By entering different values for C, determine the largest and smallest totally real z that is nice.

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For C = real, the largest nice C is _____. The smallest nice C is _____.
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By entering different values for C, determine the largest and smallest totally imaginary z that is nice.

For C = imaginary, the largest nice C is _____. The smallest nice C is _____.

4. Looking for the boundary between "nice" and "not-nice" points

You have effectively found boundary points between nice and not-nice points that can be represented as dots in the complex plane. Try to find the curve which connects these dots. (You have undoubtedly already determined that you will only need to look at the upper half complex plane for this. **By experimenting with your spreadsheet, determine a bunch of boundary points and connect them in the complex plane.** For example, based on the examples you tried in part 2, you would guess that (.25, 0) and (-.375,.595) are boundary points. Here's what they look like:



Nice - Not Nice Boundary

5. The boundary and much more: FRACTINT

Play around as much as you like with the Mandelbrot Set fractal using FRACTINT. Zoom around the plane. How many mandelbugs within mandelbugs can you find? Stake out your favorite place in the Mandelbrot Terrain (and remember how to get there.) Experiment with color cycling. Enjoy.

Sketch your favorite picture:



Where is this in the complex plane?

6. The Julia Set

Every now and then, with an exquisite Mandelbrot image in front of you, hit the Spacebar once to see the Julia set associated with the center point in the Mandelbrot image. You may zoom and color cycle with these pictures also. Toggle back for the Mandelbrot picture.