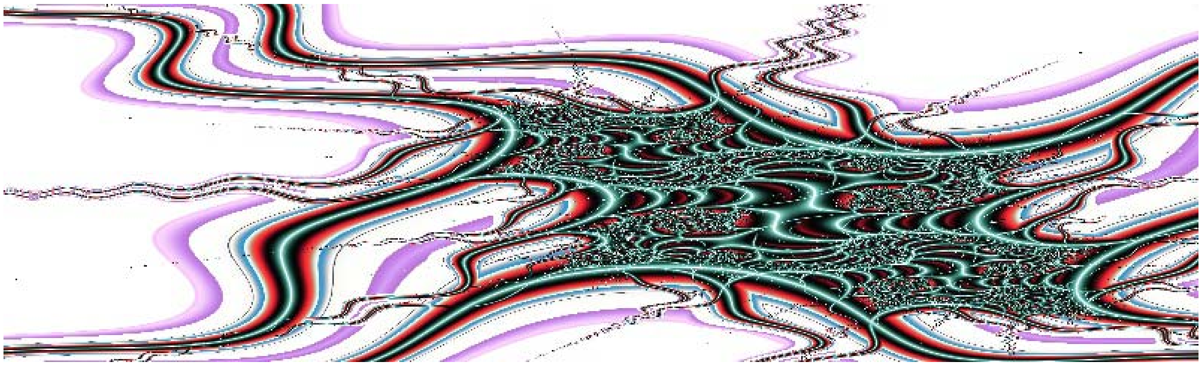

Classroom Chaos 5

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0. Population Modeling

Population modeling using discrete maps is often succinctly summarized in the following way:

$$P_{t+1} = f(P_t; r)$$

Here P represents populations at two successive “time-steps”, *r* is an adjustable parameter (usually representing a combination of birth and death rates, and *f* is the function which links today’s population with tomorrow’s. Starting with an initial population and fixed birth parameter *r*, the map generates a “time-series” of population predictions. (Note: Map output is sometimes referred to as the “orbit” of the initial value.) Here are two examples:

Linear growth/decay: $P_{t+1} = r\, P_t$

Logistic Growth: $P_{t+1} = r\, P_t\,(1 - P_t)$

1. Population Dynamics: Linear Growth Function

a. Implement the linear growth model in the following way:

	A	B
1	P_zero	.7
2	R	.45
3		
4		= B1
5		= B2 * B4

Copy Cells B5 down through B54

b. Create a line graph of your data in column B. Tile your windows so that the chart and the sheet are both visible.

c. Experiment with the values of the initial population (P_zero) and growth parameter (r), and try to discover a predictive rule that *qualitatively describes the long-term behavior* of the time-series for any values of these parameters.

Your predictive rule is:

2. Population Dynamics: Logistic Growth

a. Implement the logistic growth model in the following way:

	A	B
1	P_zero	.7
2	r	.45
3		
4		= B1
5		= B2 * B4*(1-B4)
6		= B2 * B5 *(1-B5)
7		= B2 * B6 *(1-B6)

b. Copy Cells B5 down through B54

c. The logistic map as written above is designed to yield output values in the interval [0,1] given input taken from the same interval. As a first numerical experiment, find out what range of *r* values guarantees that this is true.

Your range of r-values is: ___________________________

d. Create a line graph of your data in column B. Tile your windows so that the chart and the sheet are both visible.

e. Experiment with the values of the initial population (P_zero) and growth parameter (*r*), and try to discover a *predictive rule that qualitatively describes the time-series* for any values of these parameters. In particular, can you find a "chaotic" regime?

Your predictive rule:

3. How Chaotic is it? Embedding and Return Maps

a. Another way to look at you data is to look at successive map iterates versus the previous map value. To do this, simply add another column on your logistic sheet that copies your original column shifted by one row:

	A	B	C
1	P_zero	.7	
2	r	2.4	
3			
4		= B1	=B5
5		= B2 * B4*(1-B4)	=B6
6		= B2 * B5 *(1-B5)	=B7
7		= B2 * B6 *(1-B6)	=B8

b. After copying down, make an x-y chart of column C vs. column B values. Arrange your desktop so that you can see three windows: the sheet, the time-series, and the so-called "return-map".

c. Your spreadsheet has now "embedded" your time series in a different "space", namely one now in which time is not visible. Do you see the connection between the time series view and the return-map view? What would the return map look like for the linear growth model?

3. Universality

a. Change your logistic sheet to implement the sine map:

$$P_{t+1} = r \sin(P_t)$$

Here you will have to find r values that keep P "manageable". Play around and do all the same stuff you did for the logistic map (including the Return Map).