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A new kind of science, by Stephen Wolfram, Wolfram Media, Inc., Champaign, IL, 2002, xiv+1197 pp., $44.95, ISBN 1-57955-008-8

Joseph Fourier begins his book *The Analytical Theory of Heat* [FOU] by saying,

> Primary causes are unknown to us; but are subject to simple and constant laws, which may be discovered by observation, the subject of them being the object of natural philosophy. . . . The object of our work is to set forth the mathematical laws which this element [heat] obeys. The theory of heat will hereafter form one of the most important branches of general physics.

Stephen Wolfram follows in the same exuberant spirit, as he says on page 2 of his new book that

> It took me more than a decade to come to terms with this result [that simple computer programs can produce complex behavior], and to realize just how fundamental and far-reaching its consequences are. In retrospect there is no reason the result could not have been found centuries ago, but increasingly I have come to view it as one of the more important single discoveries in the whole history of theoretical science. For in addition to opening up vast new domains of exploration, it implies a radical rethinking of how processes in nature and elsewhere work.

The author's message is reinforced in the flyleaf of the book (written by Wolfram), which says that "This long-awaited work from one of the world's most respected scientists presents a series of dramatic discoveries never before made public. . . . Stephen Wolfram shows how their results [certain computer experiments] force a whole new way of looking at the operation of our universe." Or one can look at the press releases—also written by Wolfram. One of them asserts that, "He is widely regarded as one of the world's most original scientists, as well as the most important innovator in scientific and technical computing today." Wolfram tells us that he has entered one hundred million keystrokes on his computer in the past ten years by way of creating the scientific work that we now read.[1]

Wolfram's hubris is both charming and compelling. Wolfram has many accomplishments under his belt, not the least of which is the creation of the symbol-manipulation software `Mathematica`. He won a MacArthur Prize (at the tender age of 21) for his work in the early 1980's in physics and cellular automata. The book under review is the *magnum opus* which completes and validates that early research. It tells us, in effect, that the secret of the way that the world works—from the human body to the solar system—is the little trinkets of code called cellular automata.

The book has a number of unusual features. I first noted its length—1280 pages. I was then curious to know who the publisher might be. The spine gives no information, nor the front cover, nor the title page. It turns out that the book is a *samizdat*. Which is fine—if you have the knowhow and the resources, then you can certainly publish and promote your own book. But we must not forget that

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[1]If you send twenty brief e-mail messages per day, you will find that you enter over thirty-five million keystrokes in ten years. If you do things like write books or articles, edit journals, conduct professional correspondence, or write reviews, you may find that you actually break Wolfram's keystroke record.

the traditional publishing protocol has intrinsic value. It affirms what any good writer knows: that the creation of a book is a *process*. This process involves not only a grand creative act by the author but also a prolonged and detailed interaction of the author with many editors, critics, and reviewers. Wolfram apparently chooses to cut through all this prosaic drudgery. He wishes to be the proverbial single combat warrior. The preface of the book offers no specific thanks to friends, editors, or colleagues—although he does acknowledge occasional conversations with over 300 people, many of them named. The book contains no bibliography. There are copious notes to each chapter—suggesting perhaps that Wolfram will be using the Harvard method for citations. But, no, we are disappointed again. Even when Wolfram is referring to his own work he does not give a detailed reference.

A number of years ago, the Dalai Lama visited the United States. As part of his travels, he visited the headquarters in Chicago of one of the great American news magazines. He was given the Cook's tour, and then there was a grand formal lunch at which the various executives of the enterprise pontificated *ad nauseum*. The Dalai Lama—an elfin man—sat swathed in his saffron robe, an inscrutable smile on his face, saying nothing. After about an hour, the CEO of the publishing company turned to the Dalai Lama and said, "Do you have any questions about our magazine, the nation's premiere news magazine? Go ahead, ask us anything at all." The Dalai Lama bowed his head for a moment, apparently deep in thought. Then he looked up and said, "Why do you publish it?"

There is no need to ask Stephen Wolfram *why* he published *A New Kind of Science*. He evidently has an important discovery to report, and of course he must do so. More to the point is, *why* did he not publish it in a form, and in a venue, where it will be critically read and examined by those who are qualified to evaluate its merits?[2] Why did he forsake all technicalities and water the book down to a point where it is nearly opaque? Why does it take him 1280 pages to formulate and convey his apparently rather simple message? Since Wolfram was striving for the unusual, why did he not publish the tract in Tlingit, perhaps in the form of a hologram?

In mathematics, I am accustomed to a form of discourse that involves carefully formulated statements of assertions, supported by highly structured and tightly knit reasoning[3] to support them (which we usually refer to as *proofs*). I like to see carefully chosen examples that illustrate the key ideas. I enjoy prose passages that explain what is going on behind the formalism. Stephen Wolfram eschews all these niceties. But, after all, he is offering us *A New Kind of Science*. This evidently entails a new method for formulating, presenting, and defending his thoughts.

One might recall that Isaac Newton was famous for his arrogance. But even he published his masterwork, the *Principia* [NEW], with a desultory scientific title and using a standard scientific publisher. Likewise, Carl Friedrich Gauss was no wilting flower, but he published his *Disquisitiones* [GAU] in the usual fashion—and

[2]Wolfram formerly had a contract with Addison-Wesley to publish his book. One sticking point in that relationship was that he demanded that any reviewer sign a nondisclosure agreement and promise not to study math or physics for the ensuing ten years.

[3]For a mathematician, the use of proofs that conform to the quite rigid strictures of formal mathematical reasoning is analogous to the laboratory scientist striving for *reproducible* experiments. No other scientific procedure is considered to be reliable or useful. But it must be noted that physicists are not mathematicians, and they answer to a different standard from mathematicians.

with a workaday sort of title. Similarly for Darwin [DAR]. Perhaps Wolfram comes nearest to Joseph Fourier, who struggled for a decade to get his great tract [FOU] published. He finally published it himself after he became secretary of the French National Academy of Sciences.

By publishing with a title like *A New Kind of Science*, by not using reviewers or editors, by publishing the book himself, and by rejecting all recognized and accepted forms of discourse, Wolfram in effect tells us that what he has to say is so new and so daring that he must create a new format for his ideas.[4] Of course there is a price to pay for choosing such a path. Even a well-educated and scientifically literate reader will be puzzling over definitions, poring over terminology, and battling uphill to apprehend each new thought. Wolfram, however, claims that his book is written even for the uninitiated. The reader need only know a little logic to read Wolfram's book (though see my comment below on the difference between the converse and the contrapositive).

I am afraid that I come from the old school. I was trained to believe, and I in fact do believe, that scientific ideas must speak for themselves. If they are to carry some weight and pass the test of time, then they will do so independently of whether their author wrote his first scientific paper at the age of 15, or invented Fox-Wolfram variables at the age of 19, or received his Ph.D. at the age of 20.[5] They will stand up to the tried-and-true forum of peer review. With Wolfram's ideas formulated and presented in their present incarnation, we will never know the true intrinsic merit of the work.[6] Let us now turn to the meat of Wolfram's book and see whether we can determine what he has to offer.

The crux of Wolfram's message in his new book is that cellular automata are behind all of the universe's mechanisms. Here a cellular automaton may be defined as follows:

> A simple computational mechanism that, for example, changes the color of each cell on a grid based on the color of adjacent (or nearby) cells according to a transformation rule.
>
> Ray Kurzweil [KUR]

Most of Wolfram's analysis utilizes only the simplest possible cellular automata, based on a one-dimensional line of cells, two possible colors, and rules based only on the two immediately adjoining cells. Wolfram does *not* claim to have invented cellular automata. In fact the most famous cellular automaton is *The Game of Life* by John Horton Conway, and it has existed since the late 1960's. Vestigial forms of cellular automata were around in the 1950's—even before Wolfram was born.

According to Wolfram, most cellular automata create uninteresting activity: either a constant pattern, or a repetitive pattern, or a semi-random pattern produced from a single element. These he names Class 1, 2, and 3. What he concentrates on are the Class 4 automata. These produce output that appears to be random or chaotic. This ontology is the basis for all of Wolfram's analysis, yet the discussion

[4]One thinks of Andrew Wiles, who had to virtually re-invent elliptic function theory in order to prove Fermat's Last Theorem. Yet he still had his work refereed, and he published it in the *Annals of Mathematics*.

[5]All useful information obtained from a timeline of his life and achievements supplied by Stephen Wolfram—http://www.stephenwolfram.com/scrapbook/timeline.html.

[6]Freeman Dyson is quoted as saying, "There's a tradition of scientists approaching senility to come up with grand, improbable theories. Wolfram is unusual in that he's doing this in his 40s."

of what it *means* and how the classes are defined is reminiscent of exegetical discussions about the difference between nebulae and galaxies. Unfortunately, such vagueness is the hallmark of this 1280 page monograph. Much of Wolfram's analysis concentrates on cellular automata Rule 110. Wolfram himself is particularly partial to Rule 30. In fact these rules are quite succinct (and beware that between 30 and 110 there are 79 other rules). They are cogently and explicitly and unambiguously defined. For instance, Rule 30 says (p. 27), "take the new color of the cell to be whatever the previous color of its left-hand neighbor was." It is astonishing to me that the entire function of the universe—from the creation of the Mona Lisa to my current state of dyspepsia—emanates from a rule such as this. One must read the remaining 1253 pages to find out why.

To repeat, Wolfram concentrates on Class 4: the generation of ostensibly random behavior from simple, deterministic rules. The point here is elementary but incisive: we begin with a single black square in the grid and a very simple rule of logic for replication. The result is a fantastically complicated organism of squares that one could never have predicted nor analyzed. Wolfram provides copious examples—from leopard's spots (p. 426) to insect muscle sections (p. 385) to the atmosphere of Jupiter (p. 377) to patterns in molluscs (p. 423)—of phenomena in nature that (*pictorially*) can be generated from a cellular automaton paradigm.

This is all quite fascinating; we have been hearing a similar story from Mandelbrot and the fractal people for many years (see, for instance, [PED]). One cannot help but be reminded of the difference between genomics and proteomics. It is one thing to be able to map all the genes in a living organism (say a human being). It is quite another to say what the genes do and which groups of genes control which functions and forms. Just because Wolfram can cook up a cellular automaton that seems to produce the spot pattern on a leopard, may we safely conclude that he understands the mechanism by which the spots are produced on the leopard, or *why* the spots are there, or what function (evolutionary or mating or camouflage or other) they perform?

Those of us who are fond of counting may be tempted to observe that cellular automata are, by their very definition, of exponential complexity. Just as the message of *The Bible Code* [DRO] comes as no surprise—because it is possible to use high school algebra to produce a polynomial code that will show that *any* given phrase can be found as a regular letter sequence in the text of the Bible—so it is really no great shock that virtually any visual image can be generated by *some* cellular automaton. It is charming that a cellular automaton can generate the prime numbers (as Wolfram tells us on page 640), but will it tell us how the primes are distributed? Will it prove the Riemann Hypothesis?

A particularly telling statement of Wolfram's that encapsulates his view of cellular automata in the world around us is this:

> Whenever a phenomenon is encountered that seems complex it is taken for granted that the phenomenon must be the result of some underlying mechanism that is itself complex. But my discovery that simple programs can produce great complexity makes it clear that this is not in fact correct. [Wolfram, p. 4]

Wolfram repeats the message throughout the book. It is noteworthy for the fact that he seems to be confusing the converse with the contrapositive: Does it necessarily follow, just because a simple program can produce a picture that resembles the product of some complex mechanism, that the simple program *is the same as* the complex mechanism, or that it supplants that complex mechanism? I doubt it, and nobody can offer any proof to the contrary.

The more one reads of this book, the more one is reminded of great historical/religious tracts. It may be enlightening to do a quick comparison:

> *Let us assume, then, that the planet chosen to land on is similar to the earth. I have already said that this assumption is by no means impossible. Let us also venture the supposition that the civilization of the planet visited is in about the same state of development as the earth was 8,000 years ago.*
>
> —Erich Von Däniken [DAN, p. 23]

> *No quest has been more relentlessly pursued or has been more violent. No primitive tribe, no matter how ignorant, has failed to recognize the problem as a problem, nor has it failed to bring forth at least an attempted formulation. Today one finds the aborigines of Australia substituting for a science of mind a "magic healing crystal."*
>
> —L. Ron Hubbard [HUB, p. 7]

> *I have told you the philosophy of Knowledge. Now listen! and I will explain the philosophy of Action, by means of which, O Arjuna, you shall break through the bondage of all action.*
>
> —*The Bhagavad Gita* [BHA, p. 17]

> *And at the lowest level what I expect is that even though the rules being applied are perfectly definite, the overall pattern of connections that will exist in the network corresponding to our universe will continually be rearranged in ways complicated enough to seem effectively random.*
>
> *Yet on a slightly larger scale such randomness will then lead to a certain average uniformity. And it is then essentially this that I believe is responsible for maintaining something like ordinary space—with gradual variations giving rise to the phenomenon of gravity.*
>
> *But superimposed on this effectively random background will then presumably also be some definite structures that persist through many updatings of the network. And it is these, I believe, that are what correspond to particles like electrons.*
>
> —Stephen Wolfram, *A New Kind of Science*, p. 466

> *If each man keeps his own sense of sight, the world will escape being burned up. If each man keeps his own sense of hearing, the world will escape entanglements. If each man keeps his intelligence, the world will escape confusion. If each man keeps his own virtue, the world will avoid deviation from the true path.*
>
> —Lao Tse [LAO, p. 125]

The unifying theme of these quotations is euphoria, exhortation, and monumental non-specificity. This is not the proper argot for a scientist.

One interesting, if not explicitly stated, aspect of Wolfram's work is that it seems to suggest that the world is deterministic—in the sense of Newton. Yet one of the thrusts of twentieth century physics has been that the world is *not*. The Heisenberg uncertainty principle,[7] statistical mechanics, and many other parts of the modern theory give us substantive reason to think that certain forms of physical information are unknowable. One might think that Wolfram—especially as he has worked in quantum theory—would address this apparent contradiction.

Unfortunately, it is in these considerations that Wolfram reaches an apogee of vagueness. The Wolfram quotation above, wherein I am comparing him to Lao Tse and the Bhagavad Gita, comes from his discussion of quantum mechanics. I will

[7] The Heisenberg uncertainty principle is really a triumph of human thought. Contrary to what social philosopher Carl Gustav Jung seemed to believe [JUN], it is a fact of physics that can be stated analytically. To wit, $\int x^2 f^2(x)\,dx \int p^2 \hat{f}^2(p)\,dp \geq 1$.

grant that he is trying to communicate with a broad and undifferentiated audience, but his remarks are so vague and ill-formulated that it is almost impossible to determine what he is trying to say.

Likewise Wolfram's comments about statistical mechanics are at the level of sophistication wherein one says that many things in the universe are random and statistical mechanics attempts to cut through this randomness by averaging; by contrast, the theory of cellular automata cuts through the randomness by producing a deterministic process that *appears* to be random.

It is worth noting that Raymond Kurzweil's review [KUR] provides an authoritative and detailed critique of the essential physics of Wolfram's work. This goes far beyond anything that this reviewer is able to offer, and we content ourselves with a brief quotation from [KUR]:

> In summary, Wolfram's sweeping and ambitious treatise paints a compelling but ultimately overstated and incomplete picture. Wolfram joins a growing community of voices that believe that patterns of information, rather than matter and energy, represent the more fundamental building blocks of reality.

Part of my problem with Wolfram's monumental work is my pedigree. I am a mathematician who is trained to read and evaluate mathematical discourse. I am a great believer in communication, and I respect those who can cut through the recondite terminology and formalism of their subject and just tell people (especially laymen) what is going on. Stephen Hawking, for instance, is a master of this technique (see [HAW] for a sterling example).[8] I am afraid that Wolfram needs some tutoring and refinement in the expository arts. Unfortunately, in his efforts to express his ideas without using any equations or technicalities, he devolves to a low common denominator of repetition, hand waving, and pap. His description of a cellular automaton that generates primes (p. 640) is inchoate, and his explanation of how quantum mechanics (pp. 537–45) and statistical mechanics (p. 967, for instance) fit into the cellular automata context is enigmatic at best.

In one of his many interviews and press releases, Wolfram avers that the ideas in this book will, within our lifetimes, be taught to schoolchildren. *They are that fundamental.* But think back over the past fifty years. How many ideas from physics that were developed during that period are now taught in the schools? Or even to undergraduates? Well, there are fractals. But that is an unfortunate social anomaly. It is going to be some time before scientists can digest and evaluate what Wolfram is telling us. I would guess that most physicists would insist on a more traditional and rigorous treatment of the ideas in *A New Kind of Science* before they will take them seriously. Pictures of cellular automata are all well and good. This book has 1000 of them, and they certainly serve to illustrate that a cellular automaton can create baffling complexity starting from very little. I am not yet convinced that they tell us much of anything about science.

One interesting feature of Wolfram's book is the author's claim that the creation of `Mathematica` was a stepping stone to his work on cellular automata. To wit, he could not have done the necessary calculations without the aid of this marvelous and powerful software. Fair enough. The chapter notes are full of `Mathematica`

[8]It should be noted that Hawking does not conflate his popular writing with his scientific writing. He sees the two as separate efforts, with separate functions. He knows clearly, in each case, who his audience is. Wolfram could learn a lesson here.

code. But, frankly, it is not very enlightening. What we are given (e.g., pp. 883–4, p. 889, p. 928, pp. 955–6, etc.) is a practical guide to creating empirical computer phenomena. This is not science.

In the early twentieth century there was a great vogue in phenomenological science. Perhaps the highest and finest form of that literary genre was D'Arcy Wentworth Thompson's *On Growth and Form* [THO]. Evolution theorists, reconstructive surgeons, and many others still refer to Thompson's somewhat dreamy (but altogether serious) discussions of how two fish that can be conformally mapped to each other must somehow be evolutionarily related. Other works of the time (somewhat less distinguished than Thompson's) argue that the body is like a machine, or the brain like a water faucet, or the digestive tract like a trash compactor. It seems as though there is now a new surge in this kind of anti-theoretical, power-to-the-people, damn-the-prerequisites approach to hard science. It would be disrespectful of me to suggest that that is all there is to Wolfram's new book. But there are many traits in common. *A New Kind of Science* is obviously a very serious effort to describe his idea of how the universe is in effect one big cellular automaton. I trust that Wolfram's ten-year labor is the first step of a long process to develop and validate these potentially important ideas.

At the end of the film *Chinatown*, detective Jake Gittes (played by Jack Nicholson) has uncovered a vastly complex and nefarious web of evil perpetrated by wealthy land mogul Noah Cross (played by John Huston). In the dramatic closing scene, Nicholson is trying to find words to explain to the police the infrastructure of depravity that he has identified, but he can find no way to articulate his thoughts. He ultimately points at Huston and cries, "He's rich!" Huston adopts a wry look on his face, smiles, and says, "I didn't know it was a crime to be rich . . . ," and then the credits roll. Stephen Wolfram seems to have uncovered a vastly complex and profound scheme of how the world functions. His intention is that his ideas will supersede all previous scientific thought—from Archimedes to Newton to Heisenberg to Witten. He has invested ten years and 1280 pages (and 100 million keystrokes on his computer!) in endeavoring to explain his discovery—not just to his colleagues but to the world at large. It is a noble effort, but in the end he is merely pointing his finger and crying, "It's complex!" I can just hear old Mother Nature saying, "I didn't know it was a crime to be complex."

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