

# **Physics and the Real World**

No current physics experiment or theory explains the nature—or even the existence—of emotions, money, fine art, football games, or people. What can physics say about such things?

## George F. R. Ellis

Physics is the model of what a successful science should be. It provides the basis for the other physical sciences and biology because everything in our world, including ourselves, is made of the same fundamental particles, whose interactions are governed by the same fundamental forces.

It's no surprise then, as Princeton University's Philip Anderson has noted, that physics represents the ultimate reductionist subject: Physicists reduce matter first to molecules, then to atoms, then to nuclei and electrons, and so on, the goal being always to reduce complexity to simplicity (see PHYSICS TODAY, July 1991, page 9). The extraordinary success of that approach is based on the concept of an isolated system. Experiments carried out on systems isolated from external interference are designed to identify the essential causal elements underlying physical reality.

The problem is that no real physical or biological system is truly isolated, physically or historically. Consequently, reductionism tends to ignore the kinds of interactions that can trigger the emergence of order, patterns, or properties that do not preexist in the underlying physical substratum. Biological complexity and consciousness-as products of evolutionary adaptation-are just two examples. Physics might provide the necessary conditions for such phenomena to exist, but not the sufficient conditions for specifying the behaviors that emerge at those higher levels of complexity. Indeed, the laws of behavior in complex systems emerge from, but are to a large degree independent of, the underlying low-level physics. That independence explains why biologists don't need to study quantum field theory or the standard model of particle physics to do their jobs.

Moreover, causes at those higher levels in the hierarchy of complexity have real effects at lower levels, not just the reverse as often thought. Consequently, physics cannot predict much of what we see in the world around us. If it could predict all, then free will would be illusory, the inevitable outcome of the underlying physics.

#### Levels and hierarchy

True complexity, with the emergence of higher levels of order and meaning, including life, occurs in modular, hier-

George Ellis is an emeritus professor in the department of mathematics at the University of Cape Town in South Africa. archical structures.<sup>1,2</sup> Consider the precise ordering in large intricate networks—microconnections in an integrated chip or human brain, for example. Such systems are complex not merely because they are complicated; order here implies organization, in contrast to randomness or disorder. They are hierarchical in that layers of order and complexity build upon each other,

with physics underlying chemistry, chemistry underlying biochemistry, and so forth. Each level can be described in terms of concepts relevant to its own particular structure particle physics deals with behaviors of quarks and gluons, chemistry with atoms and molecules—so a different descriptive language applies at each level. Thus we can talk of different levels of meaning embodied in the same complex structure.

The phenomenon of emergent order refers to this kind of organization, with the higher levels displaying new properties not evident at the lower levels. Unique properties of organized matter arise from how the parts are arranged and interact, properties that cannot be fully explained by breaking that order down into its component parts.<sup>3,4</sup> You can't even describe the higher levels in terms of lower-level language.

Theories such as the gas laws or Ohm's law provide a phenomenological understanding of the behavior of atoms or charges.<sup>5</sup> In particular, they are examples of laws that emerge from the particles' joint, as compared to individual, behavior. The higher, many-body levels are more complex and less predictable than the lower levels; we have reliable phenomenological laws describing behavior at the levels of physics and chemistry, for instance, but not at the levels of psychology and sociology.

### From bottom up to top down

Higher-level variables are often aggregates of lower-level variables, and determined by them. But the higher-level variables reveal important properties of the hierarchy that are otherwise hidden. An electric current that flows in a wire, for instance, can be represented at a macroscopic level by a variable that specifies the total amount of charge flowing in that wire. The amperage thus provides a useful "coarse-grained" description of the microscopic situation.

Appropriate choice of such higher-level variables is the key to a phenomenological understanding of the higher levels. The flow of current in a wire can be related to the voltage across it and resistance through it, but does not offer details about the electron distribution. That loss of lower-level information is the source of entropy. Many lower-level states could correspond to the same higherlevel state. Higher-level states are thus relatively insensitive to details of the lower-level states of a system.

What happens at each higher level is, of course, based

Figure 1. How do you explain a game of chess? At one level, the movement of charges within my hand triggers the contraction of muscle cells that prompt my arm to move a chess piece from one spot to another. But at a higher level in the hierarchy of structure and causation, I simply intend to move the chess piece: My brain instructs the muscle cells how to move particular pieces. From that perspective, my appreciation of the rules of chess determines just how those particles or charges happen to interact as I play the game. The difference underscores the influence that high levels of organization (minds interacting with other minds, say) can have over the behavior of lower levels (the interaction of atoms and charges, say) rather than just the reverse. An extraterrestrial watching the chess game, though armed with an understanding of physics, would rightly be puzzled by why certain pieces move only diagonally and others move parallel to the edges.

on the chain of causes and effects from levels below it. When I move my arm, for instance, it moves because millions of electrons attract millions of protons in my muscles. Microscopic physics underlies macroscopic effects, the cornerstone of reductionist worldviews. Laws of physics beget laws of chemistry, which beget laws of biochemistry, and so forth. One might call that bottom-up action.

But conversely, higher-level variables can control what happens at the lower levels. When I move my arm, for instance, it moves because I have decided to move it. My intention thus instructs many millions of electrons and protons to behave a certain way. The detailed structure of the hierarchical system—in this case the physiology of the nervous system—makes the movement possible (see figure 1). Such top-down action affects the nature of causality significantly, because interlevel feedback loops become possible. To appreciate the prevalence of top-down effects in the real world, consider the following examples.

**Interaction potentials.** Potentials in the Schrödinger equation, or in the action for a system, represent the summed effects from particles and forces. Therefore, they provide a way to describe, at least in principle, the nature of systems as simple as a particle in a box or as complex as computers and brains. Top-down effects occur because an ordered structure underlies the causal relations; electrons flow in specific wires that connect specific components, and specific neurons connect to other specific neurons, for instance. Moreover, externally applied potentials may represent top-down effects that the environment imposes on a system. The gravitational field generated by a massive planet alters the motions of particles measured in a laboratory on its surface, for instance.

**Nucleosynthesis.** The rates of nuclear reactions depend on the density and temperature of the interaction medium. The nuclear reactions that took place in the early universe—and hence the elements produced by nucleosynthesis at that time—depended on the universe's rate of

expansion, which is determined by macroscopic cosmological variables. The resulting nuclear abundances determine a key cosmological parameter, the average density of baryons in the universe. Similarly, the equations that determine the cosmological structure growth depend on averaged quantities such as density and expansion rate of the universe. Those quantities thus determine the formation of structure.

**Quantum measurement.** Top-down action occurs in the quantum measurement process through the collapse of the wavefunction to an eigenstate of a chosen measurement basis.<sup>6</sup> The experimenter chooses the details of the measurement—preparing the initial states, aligning axes of polarization, and so forth—and those choices determine what set of microstates can result from a measurement.

**Evolution.** The development of DNA codings—that is, the particular sequence of base pairs—occurs through an evolutionary process that results in adaptation of an organism to its ecological niche.<sup>3</sup> Consider a specific example: To adapt to polar environments, a polar bear has genes for white fur, whereas to adapt to the Canadian forest, a Canadian bear has genes for brown fur. The environments in which the two species live account for differences in the detailed DNA coding, a classic case of top-down action from environment to microstructure. There would be no way to predict the DNA coding from biochemistry alone.

**Mind on the world.** Let's say someone has a plan in mind—a proposal to build a bridge, maybe. Enormous numbers of protons, neutrons, and electrons that make up the sand, concrete, and bricks get moved around in a way that fits the plan, if implemented. The results of plans and intentions of all kinds have real effects on the world. One has only to consider how much influence carbon emissions from factories, cars, and jumbo jets have on Earth's atmosphere and the global climate.<sup>7</sup>

Concepts for things like bridges and jumbo jets may be worked out rationally through the collaboration of structural engineers, metallurgists, designers, and others. Such concepts are not the same as brain states, for they can be represented in a host of different ways—in words, diagrams, writing, or computer-aided designs, to name a few. Concepts are abstract but nevertheless determine the nature of certain objects in the world: They guide manufacture and implementation of technology, for instance.

Emotions can be as effective as rationality in influencing behavior. Plans for a bridge or jet might never leave the blueprint stage were it not for passionate advocates who inspire the community and investors who can make the plan a reality. And emotions also underlie brain development and intellect, setting up implicit goals in the developing brain. The goals can then guide neural development by acting as a value system—so-called neural Darwinism.<sup>8</sup>

Similarly, social constructions drive what happens in our everyday lives: Rules and regulations govern health care systems, housing policy, and how games such as football and chess are played. Money, another convention whose effectiveness is based entirely on social agreement, is vital for constructing bridges, jumbo jets, and most other manufactured objects in our world.

Causal models of the real world remain incomplete unless they account for the various effects of intention, purposes, and goals. Multiple top-down actions allow various causal chains in higher levels of a complex system to coordinate action at lower levels in a coherent way. Because of the effectiveness of human minds at controlling lower levels of structure, the causal hierarchy bifurcates, distinguishing causation that involves choices and intention from causation that does not (see figure 2).

#### Feedback control systems

The central feature of organized action is feedback control: Setting specific goals prompts specific actions designed to achieve those goals.<sup>9</sup> The simple example of a comparator—sending an error message to a controller to adjust any difference between the system state and its goal— illustrates the concept (see figure 3).

Living systems are goal seeking, of course. But the crucial issue is what determines those goals and where they come from. Numerous systems in all living cells, plants, and animals automatically, without conscious guidance, maintain homeostasis through multiple feedback mechanisms. Using enzymes, antibodies, and regulatory circuits of all kinds, our physiological systems fight intruders and control breathing, heart rate, body temperature, blood pressure, and so forth.9 These processes developed historically and were determined in a particular environmental context through evolution. Not only are the feedback control systems themselves emergent, but the implied goals are emergent properties that guide numerous physical, chemical, and biochemical interactions in purposeful ways. They embody biological information that guides the development of plants and animals.<sup>10</sup>

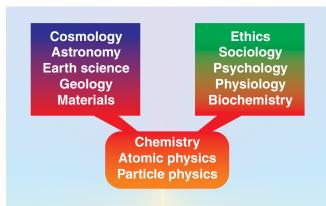
In animals, it is in the conscious choice and implementation of goals that explicit information processing comes into play. Conscious and unconscious processing of information from the senses controls purposeful action. At the highest levels, the power of symbolic abstraction, codified into language, drives analysis and understanding of the world.

Such symbolic abstraction underpins social creations such as the monetary system, mathematics, and the theories of physics. These are all emergent phenomena. Although the theories of physics, for instance, are nonphysical, they largely determine the development of technologies. While physics theories can be understood as concepts in the brain, they are certainly not brain states and don't exist in the same way physical objects do. Rather, concepts, ideas, and information exist independently of any specific representation; they can be represented in books, CDs, computer memory, or the spoken word.

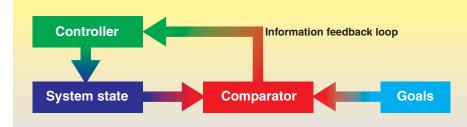
The key point about causality in this real-world context is that multiple causality (interlevel, as well as intralevel) is always operating in complex systems. Thus one can have top-down, bottom-up, and same-level system "explanations," all applicable simultaneously.

Analysis explains the properties of a system through the behavior of its component, lower-level parts. Systems thinking, in contrast, tries to understand the properties of an interconnected, complex whole,<sup>11</sup> and explains the properties of an entity through its role in relation to higher levels in the system's structure. To appreciate the distinction, one can answer the question, "Why is an aircraft flying?" in different ways.

In bottom-up terms, it flies because air molecules move at different speeds over the top and bottom wing surfaces to create a pressure difference that lifts the plane against gravity-Bernoulli's principle. In same-level terms, the airplane flies because the pilot is flying it, after a rigorous training and testing process to develop the requisite skills, and because the airline's timetable, let's say, dictates a scheduled flight. And, in top-down terms, an airplane flies because it was designed to fly! A team of engineers at some point worked within a historical context of the development of metallurgy, combustion, lubrication, aeronautics, machine tools, computer-aided design, and so forth to design the thing. All this occurs in the economic context of a society with a transportation need and the industrial infrastructure to mobilize the necessary manpower and resources to actually manufacture airplanes. A brick does not fly because it was not designed to fly.



**Figure 2. This list of academic disciplines** represents a hierarchy of causal relations. Each level underlies what happens at each subsequent higher level in the sense that principles of organization, or laws, in one field can themselves organize into new principles: The laws of electron and atomic motion beget laws of thermodynamics and chemistry, which in turn beget laws of crystallinity and plasticity or laws regulating how biomolecules form and interact, and so forth.<sup>4</sup> At a certain point up the hierarchy, the organizing principles differ fundamentally: those principles that involve unconscious natural systems (left) and those that involve conscious human choices (right). The highest level of intention—ethics—not only influences what is done, but addresses the question of what ought to be done.



**Figure 3. The basic feedback control process.** The comparator determines the difference between the system state and the goal: Feedback from an error signal activates the controller to correct the mismatch between the system's state and its programmed goal. Such simple circuits are ubiquitous—controlling the heat of a shower, the direction of an automobile, or speed of an engine—and analogous to how goals, including those in our brains, become effective. The outcome is determined by the goals rather than the initial data.

All those explanations are simultaneously true; otherwise, the plane would not be flying. The higher-level explanations rely on the existence of the lower-level explanations to make sense, but they are clearly of a different nature. Moreover, those high-level explanations are not reducible to lower-level causes, nor dependent on their specific nature. The bottom-up explanation would not apply if the higher-level explanations, the product of human intention, had not created a situation that made a bottomup causal chain relevant. And the higher-level decisions would never have been made if lower-level interactions disallowed flying.

## The limits of physics

Present-day physics includes nothing of our deliberate intention to create objects like airplanes or games like chess. Indeed, no current physics theory or experiment explains the nature or even the existence of musical symphonies, chess matches, teapots, or jumbo jets. Even if we were to attain a comprehensive theory of fundamental interactions, physics would still fail to address human purpose and hence would provide a causally incomplete description of the real world around us.

Could today's physics ever be extended to actually incorporate such features? The minimal requirement would be to extend physical theory to include relevant higherlevel variables-as happened when appropriate coarsegrained higher-level variables such as entropy, specific heat, and so forth were introduced in the 19th century to explain macroscopic physical effects. To account for human purpose, one would have to include some kind of consciousintention function  $\Psi$ , dependent on lower-level variables, that would, at least in principle, cover higher-level mind effects. One would then look for mathematical equations that reliably predict the evolution of this function, or at least show how it arises from the lower-level variables. I suspect that most physicists would regard such an ambitious project as lying outside the proper scope of their work. In any case it would be too complex to be practical.

However, there is another aspect to consider—basic principle. Brains are networks of neural cells, a fact that prompts some to claim there's nothing in principle to stop us from fully understanding them. One just needs to know enough about the state of the brain and the person's stored memories to apply physics and predict future behavior. There is no evidence that the mind is free of biological and physical determinism. Taken to its extremes, this view argues that although the universe is immensely complicated, it can be thoroughly comprehended through bottom-up causation alone. Predicting human intentionality is difficult only because we don't know enough about brains to make the calculation. Physics is all there is.

Despite its appeal to some, this kind of claim is in fact an unprovable philosophical supposition about the nature of causation; the claim is without predictive power—that is, no observable consequences follow from it—and without experimental support. Everyday experience suggests that such a belief is wrong.<sup>12</sup> The key issue is whether the higher levels in the hierarchy have real autonomous causal powers, independent of the lower levels, and can control

their context; or whether all causal powers reside at the lower levels while higher levels dance to their algorithmic tune and merely *appear* to have autonomy.

The implied claim in the cosmological context is that the particles that existed in the early universe just happened to be positioned so precisely that they made it inevitable that 14 billion years later human beings would exist, Francis Crick and James Watson would discover the structure of DNA, Charles Townes would conceive of the laser, and Edward Witten would develop M-theory.

That is patently absurd. It is inconceivable that truly random quantum fluctuations in the inflationary era of the universe could have uniquely implied the future inevitability of the *Mona Lisa*, Horatio Nelson's victory at Trafalgar, and Albert Einstein's theory of relativity. Such later creations of the mind are clearly not random. On the contrary they exhibit high levels of organization that embody sophisticated understanding of painting, military tactics, and physics, which cannot have arisen directly from random initial data.

Furthermore, the early universe perturbations could not have been structured to intentionally produce those later outcomes. Apart from the incredibly fine tuning required to make it happen, quantum uncertainty and the existence of chaotic systems that affect human life and biological evolution would prevent such a Laplacian mechanical prediction from working out. The necessary detailed predictability from the bottom up is unattainable, even in principle.<sup>13</sup>

Far more likely is that conditions in the early universe led to the eventual development of minds that—by virtue of their precisely ordered structure—are as autonomously effective as they seem to be and can create higher-level order without any fine dependence on lower-level physics. Coarse-graining in the brain relates higher-level variables to lower-level ones, and feedback control implements higher-level goals; both features damp out the effects of lower-level statistical fluctuations and of quantum uncertainty.

Predicting probable outcomes of the workings of the brain would be possible only if we were to take into account the higher-level entities that shape its outcomes—including abstractions such as the value of money, the rules of chess, local social customs, and socially accepted ethical values. These kinds of concepts influence what happens in the world but are not physical variables—they all lie outside the conceptual domain of physics, and have only come into existence as emergent entities within the past few thousand years.

Furthermore, you cannot understand or predict

a mind's behavior without taking into account its interaction with other minds. You cannot even know what aspects of the world are relevant unless you understand the social context. So, you cannot predict the future on the basis of the lower-level structures alone; you have to also include the effects of higher-level structures. But unless you understand those structures at their own level, you don't know what aspects of the lower-level variables are relevant.

Reductive physics characterizes part of the causal nexus in operation in the workings of the brain—the bottom-up aspects—but cannot account for crucial topdown influences in operation, such as those mentioned above, that determine which of the physically possible outcomes actually occur. And above all, we should not too hastily conclude that we can understand by physics alone

what happens in the brain until we properly understand consciousness and free will. Despite some extravagant claims made by a few adventurous souls, we don't have a clue how consciousness emerges from the underlying physics. We don't even know the appropriate questions to ask.<sup>14</sup>

If physics can't account for human intentions, can it account for animal behaviors? The same argument applies: Physical conditions in the early universe cannot possibly have been finetuned enough to produce the dance of a bee or the web of a spider. One might suppose that, if fully known, the physical conditions in one instance could have been used to predict what would happen in subsequent in-

stances, right through to the dancing bee. But ever-higher levels of interactions create results that are unpredictable from the vantage point of lower levels.

Physics by itself cannot comprehend any animal behavior that is adaptive and context-dependent—beavers' dam-building, birds' nest-building, or whales' cooperative hunting. Those behaviors are made possible but not causally determined by the workings of the underlying physics and chemistry. Indeed, physics and chemistry by themselves cannot even predict the development or functioning of a single living cell, for that depends on its biological context. The cell's location in an animal and what that animal is doing, for example, can only be understood in terms of higher levels of description. The statement "the whole is greater than the sum of the parts" is truly potent in the real world.

#### **Emergent physics**

Where, then, is the cut-off point in the hierarchy, above which reductive physics cannot predict behavior? Jean-Marie Lehn argues that the level of supramolecular chemistry is the first level at which biological information becomes effective and adaptive evolution is possible.<sup>15</sup> At and above that level, historical and biological contexts are the main determinants of what actually happens in living systems. For example, the detailed sequence of bases in a strand of DNA cannot be predicted by physics alone; the higher-level evolutionary context is a key determinant, which in the case of human DNA includes crucial cultural aspects such as the development of symbolic understanding.

We should also recognize that the enterprise of science itself does not make sense if our minds cannot rationally choose between theories on the basis of the available data. A reasoning mind able to make rational choices is a prerequisite for the discipline of physics to exist.

As you go deeper in the hierarchy of complexity, the essential issue is not that the messiness of nature gets in the way of deciphering the cause–effect chain, or that processes can no longer be isolated from the world. The point is that higher-level properties themselves, including abstract theories and social constructs, are key variables in the causal chain. Paradoxically, although the higherlevel properties emerge from the lower-level processes,

> they have a degree of causal independence from them: Higher-level processes operate according to their own higher-level logic. Physics makes possible, but does not causally determine, the higher-order layers. It cannot replace psychology, sociology, politics, and economics as autonomous subjects of study because complex objects like human beings are the product of principles of organization and collective behavior that cannot be meaningfully reduced to the behavior of their component parts.<sup>3,4</sup>

## The technical challenge

The technical challenge for physicists is to see how all this relates to existence and uniqueness theorems in physics.<sup>16</sup> These theorems offer theoretical support for

the belief that physics can, in fact, provide a complete causal description of all that happens, once we are given sufficient initial data. The problem is that such theorems are not applicable to real physical systems in several ways.

What happens at the microlevel is determined by probabilistic equations, or more precisely, by one set of deterministic equations that governs the evolution of the wavefunction, along with a measurement process whose outcome is determined in a probabilistic way.<sup>6</sup> Thus, our ability to predict the future on the microscopic scale quickly diminishes as quantum uncertainties accumulate and the probability of determining possible outcomes rapidly becomes negligible. The vast majority of those alternative outcomes are predicted with equal probabilities, and thus give no useful information, such as whether prices will rise or fall on the New York Stock Exchange.<sup>13</sup>

Moreover, chaotic systems exist in significant biological contexts—the physical processes governing weather on Earth, for example. Because initial conditions can never be known at the required level of accuracy, predictability is not attainable. Only by ignoring quantum fluctuations can one contemplate that a system may be deterministic in principle despite its unpredictability in practice. But quantum randomness ensures that initial conditions cannot be prescribed, even in principle, to indefinite accuracy. Thus chaotic systems act as amplifiers of quantum uncertainty. That makes predicting the evolution of life all the



Fourteen billion years after the big bang.



more unlikely given how significantly climate and weather affect animal survival probabilities.

However, neither chaotic processes nor the probabilistic nature of quantum theory is critical to what we are discussing here, unless quantum uncertainty is somehow directly linked to how minds work. The important point is that the equations of state usually assumed in theorems about existence and uniqueness are so highly simplified that they are just not relevant to the kinds of complex hierarchical structuring that occurs in biological systems. Moreover, those equations are based on equilibrium conditions in closed systems. Consequently, they cannot account for top-down action in a hierarchy with coarse-graining of variables, feedback control loops, and the unique properties that can emerge by virtue of organization at high levels. The challenge is to derive equations that adequately represent causation in complex systems, and then to see how they can allow novel features to emerge that were not, in fact, uniquely implied by the initial data.

The usual uniqueness theorems will presumably not apply to complexity in the world because the properties that emerge from collective behavior are not implicitly coded into the initial data in the early universe. An essential role in the emergence of genuine complexity will presumably be played through Darwinian processes of natural selection that result in the accumulation of order and information as hierarchical modular structures develop. We can indeed understand those processes scientifically, provided we include the higher level effects appropriately. How that works in physical terms—what effective equations relate to what variables in the context of complex systems and what the properties of those equations are—is the real challenge in understanding complexity.<sup>17</sup>

#### References

- G. F. R. Ellis, in *The Re-Emergence of Emergence*, P. Clayton, P. C. W. Davies, eds., Oxford U. Press, New York (in press); also available at http://www.mth.uct.ac.za/~ellis/emerge.doc.
- G. Booch, Object Oriented Analysis and Design with Applications, 2nd ed., Benjamin Cummings, Redwood City, CA (1994).
- N. A. Campbell, *Biology*, Benjamin Cummings, Menlo Park, CA (1996).
- 4. R. B. Laughlin, A Different Universe: Reinventing Physics from the Bottom Down, Basic Books, New York (2005).
- S. Hartmann, Stud. Hist. Philos. Mod. Phys. 32, 267 (2001).
  R. Penrose, The Emperor's New Mind, Oxford U. Press, New York (1989).
- 7. H. J. Schellnhuber, Nature 402, C19 (1999).
- G. F. R. Ellis, J. Toronchuk, in *Consciousness and Emotion:* Agency, Conscious Choice, and Selective Perception, N. Newton, R. Ellis, eds., John Benjamins, Philadelphia (2004), p. 81.
- 9. J. H. Milsum, *Biological Control Systems Analysis*, McGraw-Hill, New York (1966).
- B-O. Küppers, Information and the Origin of Life, MIT Press, Cambridge, MA (1990).
- 11. R. L. Flood, E. R. Carson, *Dealing with Complexity: An Introduction to the Theory and Application of Systems Science*, Plenum Press, London (1990).
- 12. T. Pink, Free Will: A Very Short Introduction, Oxford U. Press, New York (2004).
- J. B. Hartle, in *The Future of Theoretical Physics and Cosmology*, G. W. Gibbons, E. P. S. Shellard, S. J. Rankin, eds., Cambridge U. Press, New York (2003), p. 38.
- 14. D. J. Chalmers, *The Conscious Mind: In Search of a Fundamental Theory*, Oxford U. Press, New York (1996).
- J.-M. Lehn, Supramolecular Chemistry: Concepts and Perspectives, VCH Verlag, New York (1995).
- S. W. Hawking, G. F. R. Ellis, The Large-Scale Structure of Space-Time, Cambridge U. Press, New York (1973).
- 17. A larger, more in-depth treatise on the emergence of complexity is available at http://www.mth.uct.ac.za/~ellis/realworld.doc. ■