PHY 105 Test 3 December 2, 2005 Solutions

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1. a) The system consists of the mass, the earth, and the air. This system is isolated, but mechanical energy is not conserved due to the air drag. Considering air drag to produce an internal energy change in the system, the conservation of energy can be expressed as

$$0 = \Delta K + \Delta U_g + \Delta E_{\text{int}}$$

In this problem, the starting kinetic energy is known, as well as the increase in internal energy due to air-drag:

$$0 = (0 - \tfrac{1}{2}mv_i^2) + (mgh - 0) + 10^3\,J$$

Solving for h yields $h = \dfrac{\tfrac{1}{2}mv_i^2 - 10^3\,J}{mg} = 95.67\text{ m}$

b) On the way down, the system is the same (and still isolated). Therefore

$$0 = (\tfrac{1}{2}mv_f^2 - 0) + (0 - mgh) + 10^3\,J \text{ and } v_f = \sqrt{\frac{2(mgh - 10^3)}{m}} = 35.4\,\text{m/s}$$

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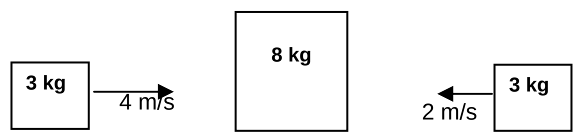
2. a) The system consists of the three masses. The only forces outside the system are the weights and the normal forces. Because these forces add up as vectors to 0, linear momentum is conserved in the collision, and the velocity of the center of mass is the same before and after the collision. (Note: air resistance is considered to be negligible.)



All movement is along the x-axis. After the **totally inelastic** collision, all masses move with the pre-collision center of mass velocity: $v_{cm} = \dfrac{3(4) + 3(-2)}{3 + 8 + 3} = 0.429\text{ m/s}$.

b) The kinetic energies before and after the collision are $K_i = \tfrac{1}{2}(3)4^2 + \tfrac{1}{2}(3)2^2 = 30\text{ J}$ and $K_f = \tfrac{1}{2}(3 + 8 + 3){v_f}^2 = 1.286\text{ J}$. The fractional kinetic energy loss is therefore

$$\frac{30 - 1.286}{30} = 95.7\%$$

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3. a) The system consists of the three masses. The only forces outside the system are the weights and the normal forces. Because these forces add up as vectors to 0, linear momentum is conserved in this collision. (Note: air resistance is considered to be negligible.)

Considering the motion of the masses to be along the x-axis, and m_1 to be the mass on the left, the momentum conservation equation is:

$$2.5(6) + 1.1(3) = 2.5v_{1f} + 1.1v_{2f}\text{, or}$$

$$2.5v_{1f} + 1.1v_{2f} = 18.3 \qquad [\textbf{Eq.1}]$$

The relative velocity of m_2 with respect to m_1 before the collision is $v_{rel,i} = 3 - 6 = -3$ m/s.
Therefore, after the **totally elastic** collision, the relative velocity switches and

$$v_{2,f} - v_{1,f} = 3 \qquad \textbf{[Eq. 2]}$$

Solving the linear system of Equations 1 and 2 yields $v_{1,f} = 4.1\overline{6}$ m/s ; $v_{2,f} = 7.1\overline{6}$ m/s

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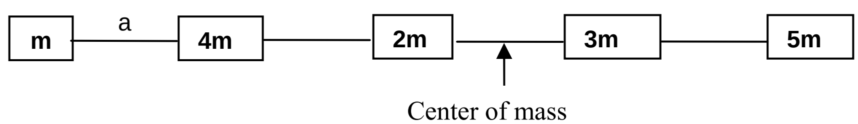
4. Let the origin of the x-axis be at the center of mass of the leftmost mass. Then, the x-position of the center of mass is given by:

$$x_{cm} = \frac{m(0) + 4m(a) + 2m(2a) + 3m(3a) + 5m(4a)}{m + 4m + 2m + 3m + 5m} = \frac{37}{15}a = 2.4\overline{6}a$$

This position is approx. halfway between 2m and 3m



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5. a) With $p_x = t^3 + 2t - 1$ kg m/s , $v_x = \frac{p_x}{m} = \frac{1}{2}\left(t^3 + 2t - 1\right)$ m/s . Therefore,

$$v_x(0) = -0.5 \text{ m/s and } v_x(3) = 16 \text{ m/s}$$

b) The average acceleration is the change in velocity divided by the time over which the change happens::

$$\bar{a}_x = \frac{16 - (-0.5)}{3} = 5.5 \text{ m/s}^2$$

c) Newton's formulation of the 2nd Law is $\sum F_x = \frac{dp_x}{dt}$. Therefore,

$$\sum F_x = \frac{d}{dt}\left(t^3 + 2t - 1\right) = 3t^2 + 2 \text{ N}$$

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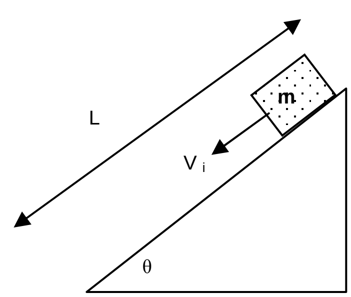
6. a) The system consists of the mass, the plane, and the earth. This system is isolated, but mechanical energy is not conserved due to the friction. Friction produces a change in thermal energy within the system, and the conservation of energy can be expressed as



$$0 = \Delta K + \Delta U_g + \Delta E_{Th}$$

In this problem, the mass loses kinetic and gravitational potential as it comes down the plane:

$$0 = \left(0 - \tfrac{1}{2}mv_i^2\right) + \left(0 - mgL\sin\theta\right) + \Delta E_{Th} \text{ and therefore}$$

$$\Delta E_{Th} = \tfrac{1}{2}mv_i^2 + mgL\sin\theta$$