Halliday/Resnick/Walker 7e

Chapter 2

2. Huber's speed is

 $v_0 = (200 \text{ m})/(6.509 \text{ s}) = 30.72 \text{ m/s} = 110.6 \text{ km/h},$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 =(110.6 + 19.0)=129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200 \text{ m}}{36 \text{ m/s}} = 5.554 \text{ s}.$$

3. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v t_c$.

(a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time interval is

$$t_1 = \frac{(40 \text{ km})}{(30 \text{ km}/\text{ h})} = 1.33 \text{ h}.$$

During the second part the displacement is $\Delta x_2 = 40$ km and the time interval is

$$t_2 = \frac{(40 \text{ km})}{(60 \text{ km}/\text{ h})} = 0.67 \text{ h}.$$

Both displacements are in the same direction, so the total displacement is

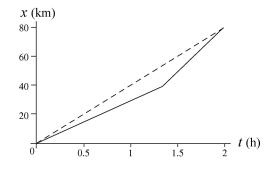
$$\Delta x = \Delta x_1 + \Delta x_2 = 40 \text{ km} + 40 \text{ km} = 80 \text{ km}.$$

The total time for the trip is $t = t_1 + t_2 = 2.00$ h. Consequently, the average velocity is

$$v_{\rm avg} = \frac{(80 \text{ km})}{(2.0 \text{ h})} = 40 \text{ km/h}.$$

(b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) As shown below, the graph consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(t_1, x_1) = (1.33 \text{ h}, 40 \text{ km})$ and the second having a slope of 60 km/h and connecting (t_1, x_1) to (t, x) = (2.00 h, 80 km). From the graphical point of view, the slope of the dashed line drawn from the origin to (t, x) represents the average velocity.



5. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write

$$x = (3 \text{ m/s})t - (4 \text{ m/s}^2)t^2 + (1 \text{ m/s}^3)t^3$$
.

We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

- (a) Plugging in t = 1 s yields x = 3 4 + 1 = 0.
- (b) With t = 2 s we get $x = 3(2) 4(2)^2 + (2)^3 = -2$ m.
- (c) With t = 3 s we have x = 0 m.
- (d) Plugging in t = 4 s gives x = 12 m.

For later reference, we also note that the position at t = 0 is x = 0.

(e) The position at t = 0 is subtracted from the position at t = 4 s to find the displacement $\Delta x = 12$ m.

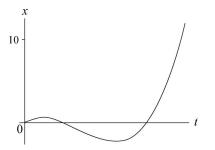
(f) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement $\Delta x = 14$ m. Eq. 2-2, then, leads to

$$v_{\rm avg} = \frac{\Delta x}{\Delta t} = \frac{14}{2} = 7 \text{ m/s}.$$

(g) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at (t, x) = (2, -2) to the highest point shown (at t = 4 s) which would represent the answer for part (f).

HRW 7e



9. Converting to seconds, the running times are $t_1 = 147.95$ s and $t_2 = 148.15$ s, respectively. If the runners were equally fast, then

$$s_{\text{avg}_1} = s_{\text{avg}_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

12. We use Eq. 2-4. to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt} (4 - 12t + 3t^2) = -12 + 6t.$$

Thus, at t = 1 s, the velocity is v = (-12 + (6)(1)) = -6 m/s.

(b) Since v < 0, it is moving in the negative x direction at t = 1 s.

(c) At t = 1 s, the speed is |v| = 6 m/s.

(d) For 0 < t < 2 s, |v| decreases until it vanishes. For 2 < t < 3 s, |v| increases from zero to the value it had in part (c). Then, |v| is larger than that value for t > 3 s.

(e) Yes, since v smoothly changes from negative values (consider the t = 1 result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that v = 0 when t = 2 s.

(f) No. In fact, from v = -12 + 6t, we know that v > 0 for t > 2 s.

13. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for t = 2.00 s and t = 3.00 s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\rm avg} = \frac{\Delta x}{\Delta t} = \frac{50.25 \text{ cm} - 21.75 \text{ cm}}{3.00 \text{ s} - 2.00 \text{ s}}$$

which yields $v_{avg} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time t = 2.00 s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

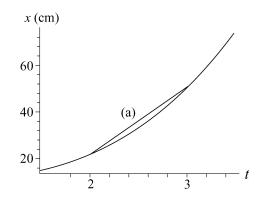
(d) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \implies t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this *x*-vs-*t* plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



14. We use the functional notation x(t), v(t) and a(t) and find the latter two quantities by differentiating:

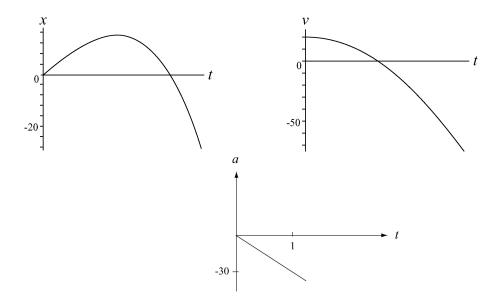
$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20$$
 and $a(t) = \frac{dv(t)}{dt} = -30t$

with SI units understood. These expressions are used in the parts that follow.

(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

- (b) From 0 = -30t, we find a(0) = 0 (that is, it vanishes at t = 0).
- (c) It is clear that a(t) = -30t is negative for t > 0
- (d) The acceleration a(t) = -30t is positive for t < 0.

(e) The graphs are shown below. SI units are understood.



15. We represent its initial direction of motion as the +x direction, so that $v_0 = +18$ m/s and v = -30 m/s (when t = 2.4 s). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30) - (+18)}{2.4} = -20 \text{ m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

16. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right) \cdot e^{-t} + (19t) \cdot \left(\frac{de^{-t}}{dt}\right)$$

If a concern develops about the appearance of an argument of the exponential (-t) apparently having units, then an explicit factor of 1/T where T = 1 second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1-t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when t = 1 s.

Now that we know *when* it stops, we find out *where* it stops by plugging our result t = 1 into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find x = 5.9 m.

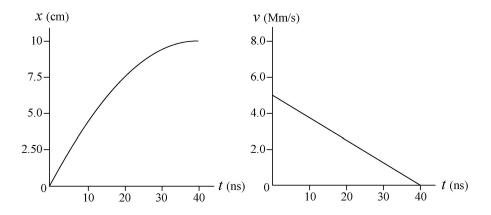
20. The constant-acceleration condition permits the use of Table 2-1.

(a) Setting v = 0 and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{5.00 \times 10^6}{-1.25 \times 10^{14}}\right) = 0.100 \text{ m}.$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.



21. We use $v = v_0 + at$, with t = 0 as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* t = 0, we set t = -2.5 s. Thus, Eq. 2-11 gives

$$v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2) (-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, t = +2.5 s and we find

$$v = (9.6 \text{ m/s}) + (3.2 \text{ m/s}^2) (2.5 \text{ s}) = 18 \text{ m/s}.$$

23. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

HRW 7e

Chapter 2

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10} (3.0 \times 10^8 \text{ m/s})}{9.8 \text{ m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2} \left(9.8 \text{ m/s}^2 \right) \left(3.1 \times 10^6 \text{ s} \right)^2 = 4.6 \times 10^{13} \text{ m}.$$

27. The problem statement (see part (a)) indicates that a = constant, which allows us to use Table 2-1.

(a) We take $x_0 = 0$, and solve $x = v_0 t + \frac{1}{2} a t^2$ (Eq. 2-15) for the acceleration: $a = 2(x - v_0 t)/t^2$. Substituting x = 24.0 m, $v_0 = 56.0$ km/h = 15.55 m/s and t = 2.00 s, we find

$$a = \frac{2(24.0 \,\mathrm{m} - (15.55 \,\mathrm{m/s}) (2.00 \,\mathrm{s}))}{(2.00 \,\mathrm{s})^2} = -3.56 \,\mathrm{m/s^2},$$

or $|a|=3.56 \text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

(b) We evaluate $v = v_0 + at$ as follows:

$$v = 15.55 \text{ m/s} - (3.56 \text{ m/s}^2)(2.00 \text{ s}) = 8.43 \text{ m/s}$$

which can also be converted to 30.3 km/h.

31. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ with t = 1.0 s, and then again with t = 2.0 s. This yields two equations for the two unknowns, v_0 and a. SI units are understood.

$$0.0 - (-2.0) = v_0 (1.0) + \frac{1}{2}a(1.0)^2$$

$$6.0 - (-2.0) = v_0 (2.0) + \frac{1}{2}a(2.0)^2$$

Solving these simultaneous equations yields the results $v_0 = 0.0$ and $a = 4.0 \text{ m/s}^2$.

(b) The fact that the answer is positive tells us that the acceleration vector points in the +x direction.

33. (a) We note that $v_A = 12/6 = 2$ m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at x = 28 m when t = 4 s. This must be the value of x for car B at that time; we use Eq. 2-15:

HRW 7e

Chapter 2

28 m =
$$(12 \text{ m/s})t + \frac{1}{2}a_{\rm B}t^2$$
 where $t = 4.0 \text{ s}$.

This yields $a_{\rm B} = -\frac{5}{2} = -2.5 \text{ m/s}^2$.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides t = 4 s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \frac{1}{2} a_{\rm B} t^2$$

where $a_{\rm B} = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_{\rm B})}$ is zero. In our case, it is zero – which means there is only one root. The cars are side by side only once at t = 4 s.

(c) A sketch is not shown here, but briefly – it would consist of a straight line tangent to a parabola at t = 4.

(d) We only care about <u>real</u> roots, which means $10^2 - 2(-20)(a_B) \ge 0$. If $|a_B| > 5/2$ then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \implies$ two real roots. The cars are side by side at two different times.

34. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop ($v_2 = 0$) at the next station. The velocity at the midpoint is v_1 which occurs at $x_1 = 806/2 = 403 \text{ m}$.

(a) Eq. 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \implies v_1 = \sqrt{2(1.34)(403)} = 32.9 \text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

$$x_1 = v_0 t_1 + \frac{1}{2} a_1 t_1^2 \implies t_1 = \sqrt{\frac{2(403)}{1.34}}$$

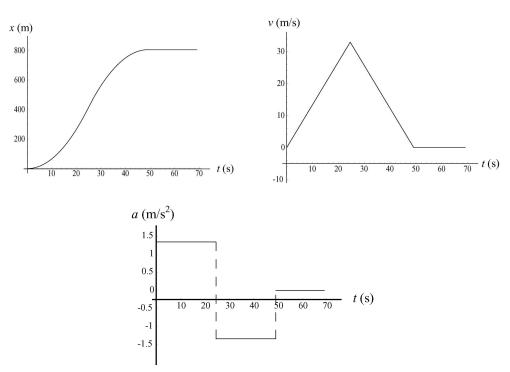
which yields $t_1 = 24.53$ s. Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain t = 49.1 s for the travel time between stations.

(c) With a "dead time" of 20 s, we have T = t + 20 = 69.1 s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\rm avg} = \frac{806 \text{ m}}{69.1 \text{ s}} = 11.7 \text{ m/s}.$$

(d) The graphs for x, v and a as a function of t are shown below. SI units are understood. The third graph, a(t), consists of three horizontal "steps" — one at 1.34 during 0 < t < 24.53 and the

next at -1.34 during $24.53 \le t \le 49.1$ and the last at zero during the "dead time" $49.1 \le t \le 69.1$).



38. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the -y direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute *t*:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that t > 0), to the result:

$$t = \frac{-12 + \sqrt{(-12)^2 - 2(9.8)(-30)}}{9.8} = 1.54 \text{ s.}$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain *v*; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

(a) Starting the clock at the moment the wrench is dropped ($v_0 = 0$), then $v^2 = v_o^2 - 2g\Delta y$ leads to

$$\Delta y = -\frac{(-24)^2}{2(9.8)} = -29.4 \text{ m}$$

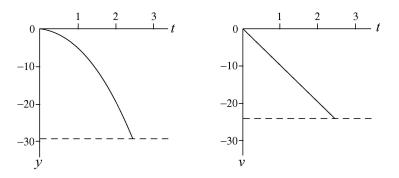
so that it fell through a height of 29.4 m.

use of Table 2-1 (with Δy replacing Δx).

(b) Solving $v = v_0 - gt$ for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24)}{9.8} = 2.45$$
 s.

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at -9.8 m/s^2 .



40. Neglect of air resistance justifies setting $a = -g = -9.8 \text{ m/s}^2$ (where *down* is our -y direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8)(-1700)} = -183$$

in SI units. Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

43. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the -y direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We are placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12 \text{ m/s}$ and that its initial coordinate is $y_0 = +80 \text{ m}$.

(a) We solve $y = y_0 + v_0 t - \frac{1}{2}gt^2$ for time, with y = 0, using the quadratic formula (choosing the positive root to yield a positive value for *t*).

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to

$$v = v_0 - gt = 12 - (9.8)(5.4) = -41$$
 m/s.

Its final speed is 41 m/s.

48. (a) With upward chosen as the +y direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_0 + at \implies 0 = v_0 - (9.8 \text{ m/s}^2)(2.0 \text{ s})$$

which leads to $v_0 = 19.6$ m/s. Now we use Eq. 2-15:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m} .$$

We note that the "2.0 s" in this second computation refers to the time interval 2 < t < 4 in the graph (whereas the "2.0 s" in the first computation referred to the 0 < t < 2 time interval shown in the graph).

(b) In our computation for part (b), the time interval ("6.0 s") refers to the 2 < t < 8 portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m},$$

or $|\Delta y| = 59 \text{ m}$.

59. We follow the procedures outlined in Sample Problem 2-8. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

(a) From Fig. 2.13a, we see that the head begins to accelerate from rest ($v_0 = 0$) at $t_0 = 110$ ms

Chapter 2

and reaches a maximum value of 90 m/s² at $t_1 = 160$ ms. The area of this region is

area =
$$\frac{1}{2}(160 - 110) \times 10^{-3} \text{s} \cdot (90 \text{ m/s}^2) = 2.25 \text{ m/s}$$

which is equal to v_1 , the speed at t_1 .

(b) To compute the speed of the torso at t_1 =160 ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region *B* has the shape of a triangle with area

area_B =
$$\frac{1}{2}$$
 (0.0600 s)(50.0 m/s²) = 1.50 m/s

From 100 to 120 ms, region C has the shape of a rectangle with area

area_c = $(0.0200 \text{ s}) (50.0 \text{ m/s}^2) = 1.00 \text{ m/s}.$

From 110 to 160 ms, region D has the shape of a trapezoid with area

area_D =
$$\frac{1}{2}$$
 (0.0400 s) (50.0 + 20.0) m/s² = 1.40 m/s.

Substituting these values into Eq. 2-26, with $v_0=0$ then gives

$$v_1 - 0 = 0 + 1.50 \text{ m/s} + 1.00 \text{ m/s} + 1.40 \text{ m/s} = 3.90 \text{ m/s},$$

or $v_1 = 3.90$ m/s.

60. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-25:

$$x_1 - x_0 = \begin{pmatrix} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$
.

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2-29 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

area_A =
$$\frac{1}{2}$$
(0.010 s) (2 m/s) = 0.01 m.

From 10 to 50 ms, region B has the shape of a trapezoid with area

area_B =
$$\frac{1}{2}$$
(0.040 s) (2 + 4) m/s = 0.12 m.

Substituting these values into Eq. 2-25, with $x_0=0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at $t_1 = 120$ ms. From 50 to 90 ms, region C has the shape of a trapezoid with area

$$\operatorname{area}_{C} = \frac{1}{2}(0.040 \text{ s}) (4+5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

area_D =
$$\frac{1}{2}(0.030 \text{ s})(5 + 7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2-25, with $x_0=0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or $x_1 = 0.50$ m.