Halliday/Resnick/Walker 7e
Chapter 3

1. The x and the y components of a vector $\vec{a}$ lying on the xy plane are given by

$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

where $a = |\vec{a}|$ is the magnitude and θ is the angle between $\vec{a}$ and the positive x axis.

(a) The x component of $\vec{a}$ is given by a_x = 7.3 cos 250° = – 2.5 m.

(b) and the y component is given by a_y = 7.3 sin 250° = – 6.9 m.

In considering the variety of ways to compute these, we note that the vector is 70° below the – x axis, so the components could also have been found from a_x = – 7.3 cos 70° and a_y = – 7.3 sin 70°. In a similar vein, we note that the vector is 20° to the left from the – y axis, so one could use a_x = – 7.3 sin 20° and a_y = – 7.3 cos 20° to achieve the same results.

4. (a) With r = 15 m and θ = 30°, the x component of $\vec{r}$ is given by $r_x = r\cos\theta$ = 15 cos 30° = 13 m.

(b) Similarly, the y component is given by $r_y = r\sin\theta$ = 15 sin 30° = 7.5 m.

5. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = 120\hat{\text{j}}$ where east is $\hat{\text{i}}$, north is $\hat{\text{j}}$, and the assumed length unit is km. Thus, we write

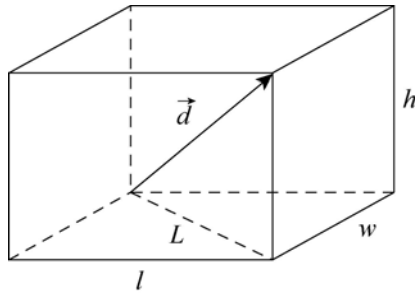
$$\vec{d}_{\text{storm}} = 100\,\hat{\text{i}}, \quad \vec{d}_{\text{new}} = A\hat{\text{i}} + B\hat{\text{j}}.$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_{\text{o}}$ readily yields A = –100 km and B = 120 km. The magnitude of $\vec{d}_{\text{new}}$ is therefore $\sqrt{A^2 + B^2} = 156\,\text{km}$.

(b) And its direction is $\tan^{-1}$ (B/A) = –50.2° or 180° + (–50.2°) = 129.8°. We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

7. The length unit meter is understood throughout the calculation.

(a) We compute the distance from one corner to the diametrically opposite corner: $d = \sqrt{3.00^2 + 3.70^2 + 4.30^2} = 6.42$.

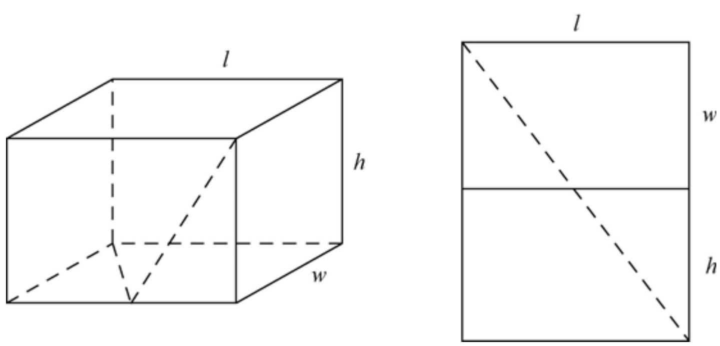


(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the *x* axis to be out of the page, the *y* axis to be to the right, and the *z* axis to be upward. Then the *x* component of the displacement is *w* = 3.70, the *y* component of the displacement is 4.30, and the *z* component is 3.00. Thus $\vec{d} = 3.70\,\hat{\mathrm{i}} + 4.30\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$. An equally correct answer is gotten by interchanging the length, width, and height.

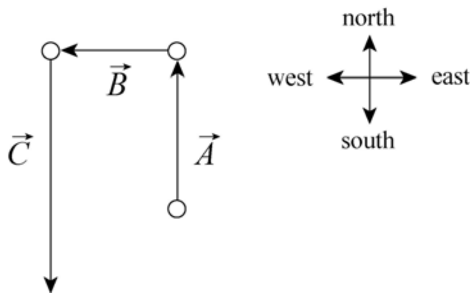


(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\min} = \sqrt{(w+h)^2 + \ell^2} = \sqrt{(3.70 + 3.00)^2 + 4.30^2} = 7.96\,\mathrm{m}\ .$$

8. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+*x* direction) and *north* as the $\hat{\mathrm{j}}$ direction (+*y* direction) All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown below:



$$\vec{A} = 3.1\,\hat{j}$$
$$\vec{B} = -2.4\,\hat{i}$$
$$\vec{C} = -5.2\,\hat{j}$$

(b) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} \;=\; -2.4\,\hat{i} - 2.1\,\hat{j}$$

whose magnitude is

$$|\vec{r}| = \sqrt{(-2.4)^2 + (-2.1)^2} \approx 3.2 \text{ km} \;.$$

(c) There are two possibilities for the angle:

$$\tan^{-1}\left(\frac{-2.1}{-2.4}\right) = 41°, \text{ or } 221°.$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +x axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

9. We find the components and then add them (as scalars, not vectors). With d = 3.40 km and θ = 35.0° we find $d \cos\theta + d \sin\theta$ = 4.74 km.

13. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as (Δx, Δy) descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives b_x = –70.0 cm.

(b) Along the y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields c_y = 80.0 cm.

(c) The magnitude of the final location (–140, –20.0) is $\sqrt{(-140)^2 + (-20.0)^2} = 141$ cm.

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the +*x* axis (172° clockwise from the +*x* axis).

17. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\vec{a} = 50\cos(30°)\,\hat{\mathrm{i}} + 50\sin(30°)\,\hat{\mathrm{j}}$$
$$\vec{b} = 50\cos(195°)\,\hat{\mathrm{i}} + 50\sin(195°)\,\hat{\mathrm{j}}$$
$$\vec{c} = 50\cos(315°)\,\hat{\mathrm{i}} + 50\sin(315°)\,\hat{\mathrm{j}}$$
$$\vec{a} + \vec{b} + \vec{c} = 30.4\,\hat{\mathrm{i}} - 23.3\,\hat{\mathrm{j}}.$$

The magnitude of this result is $\sqrt{30.4^2 + (-23.3)^2} = 38\,\mathrm{m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the +*x* direction are $\tan^{-1}$(–23.2/30.4) = –37.5°, and 180° + (–37.5°) = 142.5°. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the +*x* axis. This is equivalent to 322.5° counterclockwise from +*x*.

(c) We find

$$\vec{a} - \vec{b} + \vec{c} = [43.3 - (-48.3) + 35.4]\,\hat{\mathrm{i}} - [25 - (-12.9) + (-35.4)]\,\hat{\mathrm{j}} = 127\,\hat{\mathrm{i}} + 2.60\,\hat{\mathrm{j}}$$

in unit-vector notation. The magnitude of this result is $\sqrt{(127)^2 + (2.6)^2} \approx 1.30 \times 10^2$ m.

(d) The angle between the vector described in part (c) and the +*x* axis is $\tan^{-1}(2.6/127) \approx 1.2°$.

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d} = \vec{a} + \vec{b} - \vec{c} = -40.4\,\hat{\mathrm{i}} + 47.4\,\hat{\mathrm{j}}$, which has a magnitude of $\sqrt{(-40.4)^2 + 47.4^2} = 62\,\mathrm{m}$.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the +*x* axis are $\tan^{-1}(47.4/(-40.4)) = -50.0°$, and $180° + (-50.0°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

19. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the +*x* axis is 30° +105° = 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The *x* component of $\vec{r}$ is r_x = 10 cos 30° + 10 cos 135° = 1.59 m.

(b) The *y* component of $\vec{r}$ is r_y = 10 sin 30° + 10 sin 135° = 12.1 m.

(c) The magnitude of $\vec{r}$ is $\sqrt{(1.59)^2 + (12.1)^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the +*x* direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

23. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\,\hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts: $\vec{B}$ is 0.80 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A} + \vec{B} = 0.5\,\hat{\text{i}} + 0.8(\cos 30°\,\hat{\text{i}} + \sin 30°\,\hat{\text{j}}) = 1.19\,\hat{\text{i}} + 0.40\,\hat{\text{j}}.$$

The equation relating these is $\vec{A} + \vec{B} = \vec{C} + \vec{D}$, where

$$\vec{C} = 1.60(\cos 50.0°\,\hat{\text{i}} + \sin 50.0°\,\hat{\text{j}}) = 1.03\,\hat{\text{i}} + 1.23\,\hat{\text{j}}$$

(a) We find $\vec{D} = \vec{A} + \vec{B} - \vec{C} = 0.16\hat{\text{i}} - 0.83\hat{\text{j}}$, and the magnitude is *D* = 0.84 m.

(b) The angle is $\tan^{-1}(-0.83/0.16) = -79°$ which is interpreted to mean 79º south of east (or 11º east of south).

39. The point *P* is displaced vertically by 2*R*, where *R* is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or π*R*. Since *R* = 0.450 m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the *x* axis is horizontal and the *y* axis is vertical, the vector displacement (in meters) is $\vec{r} = \left(1.414\,\hat{\text{i}} + 0.900\,\hat{\text{j}}\right)$. The displacement has a magnitude of

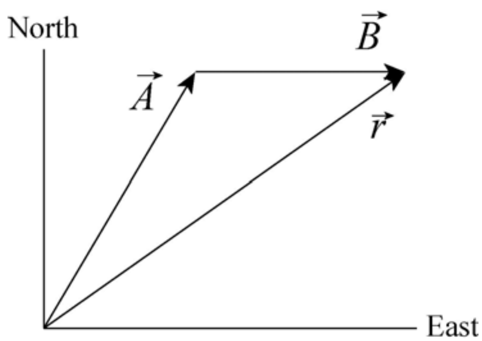
$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\,\text{m}$$

and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

51. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is 60°. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are $A_x = 250 \cos 60° = 125$ and $A_y = 250 \sin 60° = 216.5$. The components of $\vec{B}$ are $B_x = 175$ and $B_y = 0$. The components of the total displacement are $r_x = A_x + B_x = 125 + 175 = 300$ and $r_y = A_y + B_y = 216.5 + 0 = 216.5$.



(a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(300)^2 + (216.5)^2} = 370\,\text{m}.$$

(b) The angle the resultant displacement makes with the $+x$ axis is

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5}{300}\right) = 36°.$$

The direction is 36° north of due east.

(c) The total *distance* walked is $d = 250 + 175 = 425$ m.

(d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

74. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),
(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2} = 5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by *A* (with or without primes, as the case may be) and the end is indicated by *B*.