Halliday/Resnick/Walker 7e
Chapter 4

3. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k}) - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}) = -2.0\,\hat{i} + 6.0\,\hat{j} - 10\,\hat{k}$$

where the understood unit is meters.

4. We choose a coordinate system with origin at the clock center and $+x$ rightward (towards the "3:00" position) and $+y$ upward (towards "12:00").

(a) In unit-vector notation, we have (in centimeters) $\vec{r}_1 = 10\hat{i}$ and $\vec{r}_2 = -10\hat{j}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = -10\hat{i} - 10\hat{j}\ .$$

Thus, the magnitude is given by $|\Delta\vec{r}| = \sqrt{(-10)^2 + (-10)^2} = 14\ \text{cm}$.

(b) The angle is

$$\theta = \tan^{-1}\left(\frac{-10}{-10}\right) = 45°\ \text{or}\ -135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = -10\hat{i} - 10\hat{j} \rightarrow (14\ \angle\ -135°)$.

(c) In this case, $\vec{r}_1 = -10\hat{j}$ and $\vec{r}_2 = 10\hat{j}$, and $\Delta\vec{r} = 20\hat{j}$ cm. Thus, $|\Delta\vec{r}| = 20\ \text{cm}$.

(d) The angle is given by

$$\theta = \tan^{-1}\left(\frac{20}{0}\right) = 90°.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. The average velocity is given by Eq. 4-8. The total displacement $\Delta\vec{r}$ is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with $+x$ East and $+y$ North.

(a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min/h}}\right)\hat{i} = (40.0\ \text{km})\hat{i}.$$

The second displacement has a magnitude of $60.0\,\frac{\text{km}}{\text{h}}\cdot\frac{20.0\text{ min}}{60\text{ min/h}} = 20.0\text{ km},$ and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = 20.0\cos(40.0°)\,\hat{\text{i}} + 20.0\sin(40.0°)\,\hat{\text{j}} = 15.3\,\hat{\text{i}} + 12.9\,\hat{\text{j}}$$

in kilometers. And the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\text{ min}}{60\text{ min/h}}\right)\hat{\text{i}} = (-50.0\text{ km})\,\hat{\text{i}}.$$

The total displacement is

$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = 40.0\hat{\text{i}} + 15.3\hat{\text{i}} + 12.9\hat{\text{j}} - 50.0\hat{\text{i}} = (5.30\text{ km})\,\hat{\text{i}} + (12.9\text{ km})\,\hat{\text{j}}.$$

The time for the trip is (40.0 + 20.0 + 50.0) = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \left(\frac{5.30\text{ km}}{1.83\text{ h}}\right)\hat{\text{i}} + \left(\frac{12.9\text{ km}}{1.83\text{ h}}\right)\hat{\text{j}} = (2.90\text{ km/h})\,\hat{\text{i}} + (7.01\text{ km/h})\,\hat{\text{j}}.$$

The magnitude is

$$|\vec{v}_{\text{avg}}| = \sqrt{(2.90)^2 + (7.01)^2} = 7.59\text{ km/h}.$$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{7.01}{2.90}\right) = 67.5°\ \ \text{(north of east)},$$

or $22.5°$ east of due north.

7. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{(-2.0\hat{\text{i}} + 8.0\hat{\text{j}} - 2.0\hat{\text{k}}) - (5.0\hat{\text{i}} - 6.0\hat{\text{j}} + 2.0\hat{\text{k}})}{10} = (-0.70\hat{\text{i}} + 1.40\hat{\text{j}} - 0.40\hat{\text{k}})\text{ m/s}.$$

8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. All distances are in kilometers, times in hours, and speeds in km/h. The first displacement is $\vec{r}_{AB} = 483\hat{\text{i}}$ and the second is $\vec{r}_{BC} = -966\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483\text{ km})\hat{\text{i}} - (966\text{ km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483)^2+(-966)^2} = 1.08\times10^3$ km.

(b) The angle is given by

$$\tan^{-1}\left(\frac{-966}{483}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{483\hat{\mathrm{i}} - 966\hat{\mathrm{j}}}{2.25} = 215\hat{\mathrm{i}} - 429\hat{\mathrm{j}}.$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215)^2+(-429)^2} = 480$ km/h.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480 \angle -63.4°)$.

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 + 966}{2.25} = 644 \text{ km/h}.$$

9. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\mathrm{i}} + 4t^2\,\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}) = 8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}\ .$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}\,(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\ .$$

10. We adopt a coordinate system with $\hat{\mathrm{i}}$ pointed east and $\hat{\mathrm{j}}$ pointed north; the coordinate origin is the flagpole. With SI units understood, we "translate" the given information into unit-vector notation as follows:

$$\vec{r}_o = 40\hat{\text{i}} \quad \text{and} \quad \vec{v}_o = -10\hat{\text{j}}$$
$$\vec{r} = 40\hat{\text{j}} \quad \text{and} \quad \vec{v} = 10\hat{\text{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_o = -40\ \hat{\text{i}} + 40\ \hat{\text{j}}.$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-40)^2 + (40)^2} = 56.6\ \text{m}.$

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40}{-40}\right) = -45° \text{ or } 135°.$$

Since the desired angle is in the second quadrant, we pick $135°$ ($45°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_o = (56.6\angle 135°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_{\text{avg}}$ is simply the magnitude of the displacement divided by the time (Δt = 30 s). Thus, the average velocity has magnitude 56.6/30 = 1.89 m/s.

(d) Eq. 4-8 shows that $\vec{v}_{\text{avg}}$ points in the same direction as $\Delta\vec{r}$, i.e, $135°$ ($45°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_{\text{avg}} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = 0.333\hat{\text{i}} + 0.333\hat{\text{j}}$$

in SI units. The magnitude of the average acceleration vector is therefore $0.333\sqrt{2} = 0.471\ \text{m/s}^2$.

(f) The direction of $\vec{a}_{\text{avg}}$ is

$$\theta = \tan^{-1}\left(\frac{0.333}{0.333}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose $45°$, and $\vec{a}_{avg}$ points north of due east.

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{\text{i}} + [6.00 - 7.00(16)]\hat{\text{j}} = 6.00\hat{\text{i}} - 106\hat{\text{j}}$$

in meters.

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\text{i}} - 28.0t^3\,\hat{\text{j}}$$

where we have written $v(t)$ to emphasize its dependence on time. This becomes, at t = 2.00 s, $\vec{v}$ = $(19.0\hat{\text{i}} - 224\hat{\text{j}})$ m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\text{i}} - 84.0t^2\,\hat{\text{j}}$, which yields $\vec{a} = (24.0\hat{\text{i}} - 336\hat{\text{j}})$ m/s^2 at t = 2.00 s.

(d) The angle of $\vec{v}$, measured from $+x$, is either

$$\tan^{-1}\left(\frac{-224}{19.0}\right) = -85.2° \text{ or } 94.8°$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the $+x$ axis) since the signs of its components imply that it is in the fourth quadrant.

14. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{\text{i}} + 8.0\,\hat{\text{j}}\right) = \left(6.0 - 8.0t\right)\hat{\text{i}}$$

in SI units. Specifically, we find the acceleration vector at t = 3.0 s to be $\left(6.0 - 8.0(3.0)\right)\hat{\text{i}} = (-18 \text{ m/s}^2)\hat{\text{i}}.$

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{\text{i}} = 0$; we find t = 0.75 s.

(c) Since the y component of the velocity, v_y = 8.0 m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\vec{v}| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). We solve for t as follows:

$$\begin{aligned}
\text{squaring } \left(6.0t - 4.0t^2\right)^2 + 64 &= 100 \\
\text{rearranging } \left(6.0t - 4.0t^2\right)^2 &= 36 \\
\text{taking square root } 6.0t - 4.0t^2 &= \pm 6.0 \\
\text{rearranging } 4.0t^2 - 6.0t \pm 6.0 &= 0 \\
\text{using quadratic formula } t &= \frac{6.0 \pm \sqrt{36 - 4(4.0)(\pm 6.0)}}{2(8.0)}
\end{aligned}$$

where the requirement of a real positive result leads to the unique answer: t = 2.2 s.

15. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{\text{i}}\right)(3.00) + \frac{1}{2}\left(-1.00\,\hat{\text{i}} - 0.50\,\hat{\text{j}}\right)(3.00)^2 = (4.50\,\hat{\text{i}} - 2.25\,\hat{\text{j}}) \text{ m}.$$

17. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0)}{9.80}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250)(3.03) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$\left|v_y\right| = gt = (9.80)(3.03) = 29.7 \text{ m/s}.$$

18. We use Eq. 4-26

$$R_{\max} = \left(\frac{v_0^2}{g}\ \sin\ 2\theta_0\right)_{\max} = \frac{v_0^2}{g} = \frac{(9.5\text{m/s})^2}{9.80\text{m/s}^2} = 9.209\text{ m } \approx 9.21\text{m}$$

to compare with Powell's long jump; the difference from $R_{\max}$ is only ΔR =(9.21 – 8.95) = 0.259 m.

21. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with $y = -PQ$ we have $PQ = \frac{1}{2}(9.8)(0.19)^2 = 0.18$ m.

(b) From $x = v_0t$ we obtain x = (10)(0.19) = 1.9 m.

22. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

which yields h = 51.8 m for y_0 = 0, v_0 = 42.0 m/s, θ_0 = 60.0° and t = 5.50 s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)^2} = 27.4\text{ m/s}.$$

(c) We use Eq. 4-24 with $v_y = 0$ and $y = H$:

$$H = \frac{(v_0\sin\theta_0)^2}{2g} = 67.5\text{ m}.$$

25. The initial velocity has no vertical component — only an x component equal to +2.00 m/s. Also, y_0 = +10.0 m if the water surface is established as y = 0.

(a) $x - x_0 = v_xt$ readily yields $x - x_0$ = 1.60 m.

(b) Using $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we obtain y = 6.86 m when t = 0.800 s and v_{0y}=0.

(c) Using the fact that y = 0 and y_0 = 10.0, the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ leads to $t = \sqrt{2(10.0)/9.80} = 1.43$ s. During this time, the x-displacement of the diver is $x - x_0$ = (2.00 m/s)(1.43 s) = 2.86 m.

30. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0}{25.0 \cos 40.0°} = 1.15 \text{ s.}$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0 \sin 40.0°)(1.15) - \frac{1}{2}(9.80)(1.15)^2 = 12.0 \text{ m.}$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = 25.0 \sin 40.0° - (9.80)(1.15) = 4.80 \text{ m/s.}$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

34. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at $t = 2.5$ s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x = 19$ m/s.

(a) During $t = 5$ s, the horizontal motion is $x - x_0 = v_x t = 95$ m.

(b) Since $\sqrt{19^2 + v_{0y}^{\,2}} = 31$ m/s (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with t = 2.5 s, we can use $y_{max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^{\,2} - 2g(y_{max} - y_0)$, or $y_{max} - y_0 = \frac{1}{2}\left(v_y + v_{0_y}\right)t$ to solve. Here we will use the latter:

$$y_{max} - y_0 = \frac{1}{2}(v_y + v_{0y})\, t \Rightarrow y_{max} = \frac{1}{2}(0 + 24.5)(2.5) = 31 \text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

44. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\text{ m/s})^2}{25\text{ m}} = 4.0\text{ m/s}^2.$$

45. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15\text{ m})}{12\text{ s}} = 7.85\text{ m/s}$$

and his centripetal acceleration is

$$a = \frac{(7.85\text{ m/s})^2}{15\text{ m}} = 4.1\text{ m/s}^2.$$

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is $a = 4.1\text{ m/s}^2$, same as part (b).

(e) The direction is up, toward the center of the orbit.

53. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

55. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20\text{ km/h} \to 5.6\text{ m/s}$, $30\text{ km/h} \to 8.3\text{ m/s}$, and $45\text{ km/h} \to 12.5\text{ m/s}$. We choose east as the $+\hat{\mathrm{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\mathrm{c\,t}} = \vec{v}_{\mathrm{c\,g}} - \vec{v}_{\mathrm{t\,g}} = 12.5\ \hat{\mathrm{i}} - (-5.6\ \hat{\mathrm{i}}) = (18.1\ \mathrm{m/s})\ \hat{\mathrm{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3\ \mathrm{m/s})\hat{\mathrm{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\mathrm{avg}} = \frac{18.1\ \hat{\mathrm{i}} - (-8.3\ \hat{\mathrm{i}})}{2.0} = (13\ \mathrm{m/s^2})\ \hat{\mathrm{i}},$$

or $|\vec{a}_{\mathrm{avg}}| = 13\ \mathrm{m/s^2}$.

(b) The direction of $\vec{a}_{avg}$ is $+\hat{\mathrm{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\mathrm{c\,g}} = \vec{v}_{0\mathrm{c\,t}} + \vec{v}_{0\mathrm{t\,g}} = (-8.3\ \hat{\mathrm{i}}) + (-5.6\ \hat{\mathrm{i}}) = (-13.9\ \mathrm{m/s})\ \hat{\mathrm{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\mathrm{avg}} = \frac{12.5\ \hat{\mathrm{i}} - (-13.9\ \hat{\mathrm{i}})}{2.0} = (13\ \mathrm{m/s^2})\ \hat{\mathrm{i}}, \quad |\vec{a}_{\mathrm{avg}}| = 13\ \mathrm{m/s^2}$$

identical to the result of part (a).

(d) The direction of $\vec{a}_{\mathrm{avg}}$ is $+\hat{\mathrm{i}}$, or eastward.

56. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{\mathrm{i}}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\mathrm{b\,g}} = \vec{v}_{\mathrm{b\,w}} + \vec{v}_{\mathrm{w\,g}} = (14\ \mathrm{km/h})\ \hat{\mathrm{i}} + (-9\ \mathrm{km/h})\ \hat{\mathrm{i}} = (5\ \mathrm{km/h})\ \hat{\mathrm{i}}$$

Thus, the magnitude is $|\vec{v}_{\mathrm{bg}}| = 5\ \mathrm{km/h}$.

(b) The direction of $\vec{v}_{\mathrm{bg}}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{c\,g} = \vec{v}_{c\,b} + \vec{v}_{b\,g} = (-6\ \text{km/h})\,\hat{\mathrm{i}} + (5\ \text{km/h})\,\hat{\mathrm{i}} = (-1\ \text{km/h})\,\hat{\mathrm{i}}\ .$$

The magnitude is $|\vec{v}_{cg}| = 1\ \text{km/h}$.

(d) The direction of $\vec{v}_{cg}$ is $-x$, or downstream.

59. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9\ \text{m/s}}{8.0\ \text{m/s}} = 1.74$$

which yields $\theta = 60°$.

60. The destination is $\vec{D}$ = 800 km $\hat{\mathrm{j}}$ where we orient axes so that $+y$ points north and $+x$ points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{pg}$ = 400 km/h $\hat{\mathrm{j}}$. This must be the vector sum of the plane's velocity with respect to the air which has (x,y) components (500cos70º, 500sin70º) and the velocity of the air (*wind*) relative to the ground $\vec{v}_{ag}$. Thus,

$$400\,\hat{\mathrm{j}} = 500\cos 70^{\circ}\,\hat{\mathrm{i}} + 500\sin 70^{\circ}\,\hat{\mathrm{j}} + \vec{v}_{ag} \quad \Rightarrow \quad \vec{v}_{ag} = -171\hat{\mathrm{i}} - 70.0\hat{\mathrm{j}}\ .$$

(a) The magnitude of $\vec{v}_{ag}$ is $|\vec{v}_{ag}| = \sqrt{(-171)^2 + (-70.0)^2} = 185\ \text{km/h}$.

(b) The direction of $\vec{v}_{ag}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0}{-171}\right) = 22.3°\ \ (\text{south of west}).$$

69. Since $v_y^2 = v_{0_y}^2 - 2g\Delta y$, and v_y=0 at the target, we obtain

$$v_{0_y} = \sqrt{2(9.80)(5.00)} = 9.90\ \text{m/s}$$

(a) Since $v_0 \sin\theta_0 = v_{0y}$, with v_0 = 12.0 m/s, we find θ_0 = 55.6°.

(b) Now, $v_y = v_{0y} - gt$ gives t = 9.90/9.80 = 1.01 s. Thus, $\Delta x = (v_0 \cos\theta_0)t$ = 6.85 m.

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos\theta_0$ = 6.78 m/s.

71. The (x,y) coordinates (in meters) of the points are A = (15, –15), B = (30, –45), C = (20, –15), and D = (45, 45). The respective times are t_A = 0, t_B = 300 s, t_C = 600 s, and t_D = 900 s.

Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point *A*.

(a) The average velocity having the least magnitude (5.0/600) is for the displacement ending at point *C*: $|\vec{v}_{avg}| = 0.0083 \text{ m/s}$.

(b) The direction of $\vec{v}_{avg}$ is 0°(measured counterclockwise from the +*x* axis).

(c) The average velocity having the greatest magnitude ($\frac{\sqrt{15^2 + 30^2}}{300}$) is for the displacement ending at point *B*: $|\vec{v}_{avg}| = 0.11 \text{ m/s}$.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +*x*) or −63° (which is equivalent to measuring 63° *clockwise* from the +*x* axis).

88. Eq. 4-34 describes an inverse proportionality between *r* and *a*, so that a large acceleration results from a small radius. Thus, an upper limit for *a* corresponds to a lower limit for *r*.

(a) The minimum turning radius of the train is given by

$$r_{\min} = \frac{v^2}{a_{\max}} = \frac{(216 \text{ km/h})^2}{(0.050)(9.8 \text{ m/s}^2)} = 7.3 \times 10^3 \text{ m.}$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{\max} r} = \sqrt{0.050(9.8)(1.00 \times 10^3)} = 22 \text{ m/s}$$

which is roughly 80 km/h.

103. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where $v_y = \sqrt{2gh}$ and we have used Eq. 2-16 with Δx replaced with h = 20 m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8)(20)}/2 = 7.0 \text{ m/s}$.