Halliday/Resnick/Walker 7e Chapter 6

3. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push \vec{F} in the +x direction). Applying Newton's second law to the x and y axes, we obtain

$$F - f_{s, \max} = ma$$
$$F_{N} - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,max} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set a = 0 to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

(a) With $\mu_s = 0.45$ and m = 45 kg, the equation above leads to F = 198 N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing m = 45 kg with m = 28 kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

6. The greatest deceleration (of magnitude *a*) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

$$a = f_{s,max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

7. We choose +*x* horizontally rightwards and +*y* upwards and observe that the 15 N force has components $F_x = F \cos \theta$ and $F_y = -F \sin \theta$.

(a) We apply Newton's second law to the *y* axis:

$$F_N - F \sin \theta - mg = 0 \Rightarrow F_N = (15) \sin 40^\circ + (3.5)(9.8) = 44 \text{ N}.$$

With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the *x* axis:

$$F \cos \theta - f_k = ma \implies a = \frac{(15) \cos 40^\circ - 11}{3.5} = 0.14 \text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the +x (rightward) direction.

9. Applying Newton's second law to the horizontal motion, we have $F - \mu_k mg = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible). Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.4 \text{ m/s}^2$. Returning to the force equation, we find (with F = 25 N and m = 3.5 kg) that $\mu_k = 0.58$.

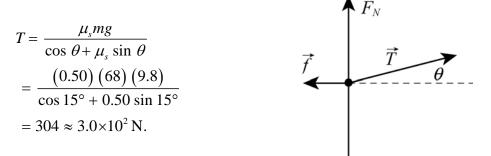
13. (a) The free-body diagram for the crate is shown below. \vec{T} is the tension force of the rope on the crate, \vec{F}_N is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and \vec{f} is the force of friction. We take the +x direction to be horizontal to the right and the +y direction to be up. We assume the crate is motionless. The equations for the x and the y components of the force according to Newton's second law are:

$$T\cos\theta - f = 0$$
$$T\sin\theta + F_N - mg = 0$$

where $\theta = 15^{\circ}$ is the angle between the rope and the horizontal. The first equation gives $f = T \cos \theta$ and the second gives $F_N = mg - T \sin \theta$. If the crate is to remain at rest, f must be less than μ_s F_N , or $T \cos \theta < \mu_s (mg - T \sin \theta)$. When the tension force is sufficient to just start the crate moving, we must have

$$T\cos\theta = \mu_s (mg - T\sin\theta).$$

We solve for the tension:



(b) The second law equations for the moving crate are

$$T\cos \theta - f = ma$$

$$F_N + T\sin \theta - mg = 0.$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - T\sin\theta$, which yields $f = \mu_k (mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k (mg - T\sin\theta) = ma$$
,

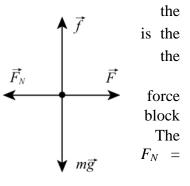
so the acceleration is

$$a=\frac{T\left(\cos\theta+\mu_k\sin\theta\right)}{m}-\mu_kg\;.$$

Numerically, it is given by

$$a = \frac{(304 \text{ N})(\cos 15^\circ + 0.35 \sin 15^\circ)}{68 \text{ kg}} - (0.35)(9.8 \text{ m/s}^2) = 1.3 \text{ m/s}^2.$$

15. (a) The free-body diagram for the block is shown below. \vec{F} is applied force, \vec{F}_N is the normal force of the wall on the block, \vec{f} force of friction, and $m\vec{g}$ is the force of gravity. To determine if block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the does not slide on the wall but if $f > \mu_s F_N$, the block does slide. horizontal component of Newton's second law is $F - F_N = 0$, so F = 12 N and $\mu_s F_N = (0.60)(12 \text{ N}) = 7.2 \text{ N}$. The vertical



component is f - mg = 0, so f = mg = 5.0 N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move f = 5.0 N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_{w} = -F_{N}\hat{i} + f\hat{j} = -(12N)\hat{i} + (5.0N)\hat{j}$$

where the axes are as shown on Fig. 6-25 of the text.

16. We find the acceleration from the slope of the graph (recall Eq. 2-11): $a = 4.5 \text{ m/s}^2$. The forces are similar to what is discussed in Sample Problem 6-2 but with the angle ϕ equal to 0 (the applied force is horizontal), and in this problem the horizontal acceleration is not zero. Thus, Newton's second law leads to

$$F - \mu_k mg = ma$$
,

where F = 40.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at $\mu_k = 0.54$.

17. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$T\cos\phi - f = ma$$
 along x axis
 $T\sin\phi + F_N - mg = 0$ along y axis

Setting a = 0 and $f = f_{s,max} = \mu_s F_N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g} \left(\sin \phi + \frac{\cos \phi}{\mu_s} \right)$$

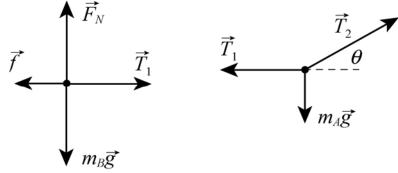
which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g} \left(\cos \phi_m - \frac{\sin \phi_m}{\mu_s} \right) = 0$$

This leads to tan $\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19^\circ$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields m = 340 kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

21. The free-body diagrams for block *B* and for the knot just above block *A* are shown next. $\vec{T_1}$ is the tension force of the rope pulling on block *B* or pulling on the knot (as the case may be), $\vec{T_2}$ is the tension force exerted by the second rope (at angle $\theta = 30^\circ$) on the knot, \vec{f} is the force of static friction exerted by the horizontal surface on block *B*, $\vec{F_N}$ is normal force exerted by the surface on block *B*, $\vec{F_N}$ is normal force exerted by the surface on block *B*, W_A is the weight of block *A* (W_A is the magnitude of $m_A \vec{g}$), and W_B is the weight of block *B* ($W_B = 711$ N is the magnitude of $m_B \vec{g}$).



For each object we take +x horizontally rightward and +y upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$T_1 - f_{s,\max} = 0$$

$$F_N - W_B = 0$$

$$T_2 \cos \theta - T_1 = 0$$

$$T_2 \sin \theta - W_A = 0$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 103$ N $\approx 1.0 \times 10^2$ N.

22. Treating the two boxes as a single system of total mass $m_{\rm C} + m_{\rm W} = 1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude 2.0 + 4.0 = 6.0 N, we apply Newton's second law (with +*x* rightward):

$$F - f_{\text{total}} = m_{\text{total}} a$$

12.0 - 6.0 = (4.0) a

which yields the acceleration $a = 1.5 \text{ m/s}^2$. We have treated *F* as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass $m_W = 3.0 \text{ kg}$) we apply Newton's second law to find the contact force *F*' exerted by the Cheerios box on it.

$$F' - f_{\rm W} = m_{\rm W} a$$

 $F' - 4.0 = (3.0)(1.5)$

This yields the contact force F' = 8.5 N.

28. (a) Free-body diagrams for the blocks *A* and *C*, considered as a single object, and for the block *B* are shown below. *T* is the magnitude of the tension force of the rope, F_N is the magnitude of the normal force of the table on block *A*, *f* is the magnitude of the force of friction, W_{AC} is the combined weight of blocks *A* and *C* (the magnitude of force \vec{F}_{gAC} shown in the figure), and W_B is the weight of block *B* (the magnitude of force \vec{F}_{gB} shown). Assume the blocks are not moving. For the blocks on the table we take the *x* axis to be to the right and the *y* axis to be upward. From Newton's second law, we have

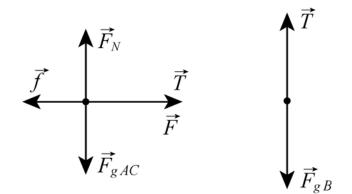
x component:
$$T-f=0$$

y component: $F_N - W_{AC} = 0$.

For block *B* take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $F_N = W_{AC}$. If sliding is not to occur, *f* must be less than $\mu_s F_N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is

$$W_{AC} = W_B / \mu_s = (22 \text{ N}) / (0.20) = 110 \text{ N}.$$

Since the weight of block A is 44 N, the least weight for C is (110 - 44) N = 66 N.



(b) The second law equations become

$$T-f = (W_A/g)a$$

$$F_N - W_A = 0$$

$$W_B - T = (W_B/g)a.$$

In addition, $f = \mu_k F_N$. The second equation gives $F_N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain

$$W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a.$$

Therefore,

$$a = \frac{g(W_B - \mu_k W_A)}{W_A + W_B} = \frac{(9.8 \text{ m/s}^2)(22 \text{ N} - (0.15)(44 \text{ N}))}{44 \text{ N} + 22 \text{ N}} = 2.3 \text{ m/s}^2.$$

32. Using Eq. 6-16, we solve for the area

$$A\frac{2m g}{C\rho v_t^2}$$

which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\rm slow}}{A_{\rm fast}} = \left(\frac{310 \text{ km/h}}{160 \text{ km/h}}\right)^2 = 3.75.$$

36. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R , where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car

and *m* is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max}=\mu_s F_N=\mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \leq \mu_s g \quad \Rightarrow \quad v \leq \sqrt{\mu_s R g}.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\text{max}} = \sqrt{\mu_s Rg} = \sqrt{(0.60)(30.5)(9.8)} = 13 \text{ m/s} \approx 48 \text{ km/h}.$$

37. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R , where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,max} = \mu_s F_N = \mu_s mg$. If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \leq \mu_s g \implies R \geq \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(8.1 \text{ m/s})^2}{(0.32)(9.8 \text{ m/s}^2)} = 21 \text{ m}.$$

39. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem 6-9, although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_{\rm s} = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With T = 6.0 s and R = 5.4 m, we obtain $\mu_s = 0.60$.

41. At the top of the hill, the situation is similar to that of Sample Problem 6-7 but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem 6-7) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37 \times 10^3 \text{ N}.$$

42. (a) We note that the speed 80.0 km/h in SI units is roughly 22.2 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal mg. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (mv^2/R)^2} = 547 \text{ N}.$$

(b) The angle is $\tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}(v^2/gR) = 9.53^{\circ}$ (as measured from a vertical axis).

45. (a) At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $F_N = 556$ N. Earth pulls down with a force of magnitude W = 667 N. The seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. Thus, he feels "light."

(b) Now F_N is the magnitude of the upward force exerted by the seat when the student is at the lowest point. The net force toward the center of the circle is $F_b - W = mv^2/R$ (note that we are now choosing upward as the positive direction). The Ferris wheel is "steadily rotating" so the value mv^2/R is the same as in part (a). Thus,

$$F_N = \frac{mv^2}{R} + W = 111 \text{ N} + 667 \text{ N} = 778 \text{ N}.$$

(c) If the speed is doubled, mv^2/R increases by a factor of 4, to 444 N. Therefore, at the highest point we have $W - F_N = mv^2/R$, which leads to

$$F_{\rm N} = 667 \text{ N} - 444 \text{ N} = 223 \text{ N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F_N = 667 \text{ N} + 444 \text{ N} \approx 1.11 \text{ kN}.$$

49. For the puck to remain at rest the magnitude of the tension force *T* of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50)(9.80)(0.200)}{1.50}} = 1.81 \text{ m/s}.$$

55. We apply Newton's second law (as $F_{push} - f = ma$). If we find $F_{push} < f_{max}$, we conclude "no, the cabinet does not move" (which means *a* is actually 0 and $f = F_{push}$), and if we obtain a > 0 then it is moves (so $f = f_k$). For f_{max} and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force equal to 556 N. Thus, $f_{max} = 378$ N and $f_k = 311$ N.

(a) Here we find $F_{\text{push}} < f_{\text{max}}$ which leads to $f = F_{\text{push}} = 222$ N.

- (b) Again we find $F_{\text{push}} < f_{\text{max}}$ which leads to $f = F_{\text{push}} = 334$ N.
- (c) Now we have $F_{\text{push}} > f_{\text{max}}$ which means it moves and $f = f_k = 311$ N.
- (d) Again we have $F_{\text{push}} > f_{\text{max}}$ which means it moves and $f = f_k = 311$ N.
- (e) The cabinet moves in (c) and (d).

56. Sample Problem 6-3 treats the case of being in "danger of sliding" down the θ (= 35.0° in this problem) incline: tan $\theta = \mu_s = 0.700$ (Eq. 6-13). This value represents a 3.4% decrease from the given 0.725 value.

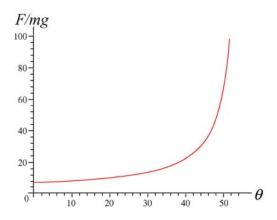
58. (a) The x component of \vec{F} tries to move the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

x direction: $F\cos\theta - f_s = 0$ y direction: $F_N - F\sin\theta - mg = 0$.

To be "on the verge of sliding" means $f_s = f_{s,max} = \mu_s F_N$ (Eq. 6-1). Solving these equations for *F* (actually, for the ratio of *F* to *mg*) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta}$$

This is plotted below (θ in degrees).



(b) The denominator of our expression (for F/mg) vanishes when

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{inf} = \tan^{-1} \left(\frac{1}{\mu_s}\right)$$

For
$$\mu_s = 0.70$$
, we obtain $\theta_{inf} = \tan^{-1} \left(\frac{1}{\mu_s} \right) = 55^\circ$.

(c) Reducing the coefficient means increasing the angle by the condition in part (b).

(d) For
$$\mu_s = 0.60$$
 we have $\theta_{inf} = \tan^{-1} \left(\frac{1}{\mu_s} \right) = 59^\circ$.

60. (a) The tension will be the greatest at the lowest point of the swing. Note that there is no substantive difference between the tension *T* in this problem and the normal force F_N in Sample Problem 6-7. Eq. 6-19 of that Sample Problem examines the situation at the top of the circular path (where F_N is the least), and rewriting that for the bottom of the path leads to

$$T = mg + mv^2/r$$

where F_N is at its greatest value.

(b) At the breaking point T = 33 N = $m(g + v^2/r)$ where m = 0.26 kg and r = 0.65 m. Solving for the speed, we find that the cord should break when the speed (at the lowest point) reaches 8.73 m/s.

70. (a) The coefficient of static friction is $\mu_s = \tan(\theta_{\text{slip}}) = 0.577 \approx 0.58$.

(b) Using

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ (with d = 2.5 m and t = 4.0 s), we obtain $\mu_k = 0.54$.

73. Replace f_s with f_k in Fig. 6-5(b) to produce the appropriate force diagram for the first part of this problem (when it is sliding downhill with zero acceleration). This amounts to replacing the static coefficient with the kinetic coefficient in Eq. 6-13: $\mu_k = \tan \theta$. Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed from what is shown in Fig. 6-5(b). Newton's second law for the uphill motion (and Eq. 6-12) leads to

$$-mg\sin\theta-\mu_k mg\cos\theta=ma.$$

Canceling the mass and substituting what we found earlier for the coefficient, we have

$$-g\sin\theta - \tan\theta g\cos\theta = a$$
.

This simplifies to $-2g\sin\theta = a$. Eq. 2-16 then gives the distance to stop: $\Delta x = -v_0^2/2a$.

(a) Thus, the distance up the incline traveled by the block is $\Delta x = v_0^2/(4g\sin\theta)$.

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in section 6-1). Sample Problem 6-3 treats the "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill): $\mu_s = \tan(\theta_{\text{repose}})$. Therefore, we expect $\theta_{\text{repose}} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

95. Except for replacing f_s with f_k , Fig 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the +*x* direction. Applying Newton's second law to the *x* axis, we have

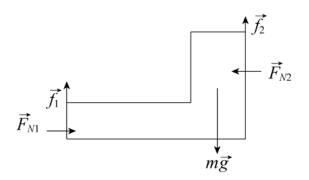
$$f_k - W \sin \theta = ma$$
 where $m = \frac{W}{g}$,

and where W = 40 N, a = +0.80 m/s² and $\theta = 25^{\circ}$. Thus, we find $f_k = 20$ N. Along the y axis, we have

$$\sum \vec{F}_{y} = 0 \Longrightarrow F_{N} = W \cos \theta$$

so that $\mu_k = f_k / F_N = 0.56$.

102. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces \vec{F}_{N1} and \vec{F}_{N2} exerted horizontally by the slabs onto her shoes and back, respectively. We will show in part (b) that $F_{N1} = F_{N2}$ so that we there is no ambiguity in saying that the magnitude of her push is F_{N2} . The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where $f_1 = \mu_{s1}F_{N1}$ and $f_2 = \mu_{s2}F_{N2}$. The problem gives the values $\mu_{s1} = 1.2$ and $\mu_{s2} = 0.8$.



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(b) We apply Newton's second law to the x and y axes (with +x rightward and +y upward and there is no acceleration in either direction).

$$F_{N1} - F_{N2} = 0$$

$$f_1 + f_2 - mg = 0$$

The first equation tells us that the normal forces are equal $F_{N1} = F_{N2} = F_N$. Consequently, from Eq. 6-1,

$$f_1 = \mu_{s1} F_N$$
$$f_2 = \mu_{s2} F_N$$

we conclude that

$$f_1 = \left(\frac{\mu_{s1}}{\mu_{s2}}\right) f_2 \; .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s1}}{\mu_{s2}}+1\right)f_2 = mg$$

which (with m = 49 kg) yields $f_2 = 192$ N. From this we find $F_N = f_2/\mu_{s2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

(c) From the above calculation, we find $f_1 = \mu_{s1}F_N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)(9.8)} = 0.60$$

or 60% of her weight.