Halliday/Resnick/Walker 7e
Chapter 8

2. (a) Noting that the vertical displacement is 10.0 – 1.5 = 8.5 m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00)(9.8)(8.5)\cos 0° = 167 \text{ J}.$$

(b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upwards understood to be the $+y$ direction).

$$\Delta U = mgy_f - mgy_i = (2.00)(9.8)(1.5) - (2.00)(9.8)(10.0) = -167 \text{ J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g$ = –167 J.

(g) With this new information (that U_0 = 100 J where y = 0) we have

$$U_i = mgy_i + U_0 = 296 \text{ J}.$$

(h) With this new information (that U_0 = 100 J where y = 0) we have

$$U_f = mgy_f + U_0 = 129 \text{ J}.$$

We can check part (f) by subtracting the new U_i from this result.

3. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the flake, so this reduces to $W = mgh$, where h is the height from which the flake falls. This is equal to the radius r of the bowl. Thus

$$W = mgr = (2.00 \times 10^{-3} \text{ kg})(9.8 \text{ m/s}^2)(22.0 \times 10^{-2} \text{ m}) = 4.31 \times 10^{-3} \text{ J}.$$

(b) The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W = -4.31 \times 10^{-3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

(d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

5. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so $\phi = 90.0°$ and $W_g = 0$ (since cos 90.0° = 0).

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = \frac{1}{2}mgh = \frac{1}{2}(825 \text{ kg})(9.80 \text{ m/s}^2)(42.0 \text{ m}) = 1.70\times10^5 \text{ J}.$$

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = (825 \text{ kg})(9.80 \text{ m/s}^2)(42.0 \text{ m}) = 3.40\times10^5 \text{ J}.$$

(d) With the reference position at C, we obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}(825 \text{ kg})(9.80 \text{ m/s}^2)(42.0 \text{ m}) = 1.70\times10^5 \text{ J}$$

(e) Similarly, we find

$$U_A = mgh = (825 \text{ kg})(9.80 \text{ m/s}^2)(42.0 \text{ m}) = 3.40\times10^5 \text{ J}$$

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

7. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20\times10^{-2} \text{ kg})(9.80 \text{ m/s}^2)(0.12 \text{ m}) = 0.15 \text{ J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.11\ \text{J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.19\ \text{J}.$$

(d) With $y = R$, at Q we have

$$U = mgR = (3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.038\ \text{J}$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.075\ \text{J}$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

9. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$.

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f. Energy conservation leads to

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\,(1 - \cos\theta)\ .$$

The speed is $v = \sqrt{2gr} = 2.08$ m/s.

(b) Since the expression for speed does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

$$0 + 196 = K_f + 29.0$$

11. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In Problem 4, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-32, we see that this is the same as U_0 which implies that $K_A = K_0$ and thus that

$$v_A = v_0 = 17.0 \text{ m/s}.$$

(b) In the solution to Problem 4, we also found $U_B = mgh/2$. In this case, we have

$$K_0 + U_0 = K_B + U_B$$
$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)$$

which leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{(17.0)^2 + (9.80)(42.0)} = 26.5 \text{ m/s}.$$

(c) Similarly,.

$$v_C = \sqrt{v_0^2 + 2gh} = \sqrt{(17.0)^2 + 2(9.80)(42.0)} = 33.4 \text{ m/s}.$$

(d) To find the "final" height, we set $K_f = 0$. In this case, we have

$$K_0 + U_0 = K_f + U_f$$
$$\frac{1}{2}mv_0^2 + mgh = 0 + mgh_f$$

which leads to $h_f = h + \frac{v_0^2}{2g} = 42.0 \text{ m} + \frac{(17.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 56.7 \text{ m}.$

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

13. We neglect any work done by friction. We work with SI units, so the speed is converted: $v = 130(1000/3600) = 36.1$ m/s.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}\,mv^2 = mgh \quad\Rightarrow\quad h = \frac{v^2}{2g} = \frac{36.1^2}{2(9.8)} = 66.5 \text{ m}.$$

If L is the length of the ramp, then L sin 15° = 66.5 m so that L = 66.5/sin 15° = 257 m. Therefore, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

16. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downwards (so $x > 0$ means it is compressed).

(a) With x = 0.190 m, Eq. 7-26 gives $W_s = -\frac{1}{2}kx^2 = -7.22 \text{ J} \approx -7.2 \text{ J}$ for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, W_s = –7.2 J.

(c) Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ mgh_0 &= -mgx + \frac{1}{2}\,kx^2 \end{aligned}$$

which (with m = 0.70 kg) yields h_0 = 0.86 m.

(d) With a new value for the height $h_0' = 2h_0 = 1.72$ m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

$$mgh_0' = -mgx + \frac{1}{2}kx^2 \;\Rightarrow\; x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

which yields x = 0.26 m.

17. We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

(a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where h = 20 m is the height of the highest point. Thus,

$$U_g = \left(5.0\times10^{-3}\ \text{kg}\right)\left(9.8\,\text{m/s}^2\right)\left(20\ \text{m}\right) = 0.98\ \text{J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is $U_s = 0.98$ J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{2(0.98\ \text{J})}{(0.080\ \text{m})^2} = 3.1\times10^2\ \ \text{N/m}\ \ = 3.1\ \ \text{N/cm}.$$

21. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_P + U_P = K_Q + U_Q$$
$$0 + mgh = \frac{1}{2}mv^2 + mgR$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is

$$F = mv^2/R = 8mg = 8(0.032\ \text{kg})(9.8\ \text{m/s}^2) = 2.5\ \text{N}.$$

and points left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the downward force of gravity

$$F = mg = (0.032\ \text{kg})(9.8\ \text{m/s}^2) = 0.31\ \text{N}.$$

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \;\Rightarrow\; mv_t^2 = mgR$$

This requires a different value of h than was used above.

$$\begin{aligned} K_P + U_P &= K_t + U_t \\ 0 + mgh &= \frac{1}{2}mv_t^2 + mgh_t \\ mgh &= \frac{1}{2}(mgR) + mg(2R) \end{aligned}$$

Consequently, $h = 2.5R = (2.5)(0.12 \text{ m}) = 0.3 \text{ m}$.

(d) The normal force F_N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for non-zero F_N — see the solution in the previous part), obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

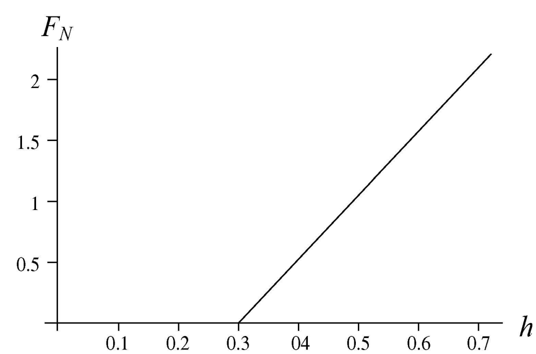
from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \Rightarrow gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ — see solution in previous part), becomes

$$F_N = \frac{2mgh}{R} - 5mg$$

Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes).



Note that for $h \leq 2.5R$, the normal force is zero.

22. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine — if it didn't break — would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where r = 18.0 m and $m = W/g = 688/9.8 = 70.2 \text{ kg}$. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh$$

where h = 3.20 m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly 9.3×10^2 N.

25. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0\,y}^2 - 2gh$ where $v_{0\,y}$ = v_0 sin 28° is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

(a) The skier starts at rest y = 20 m above the point of "launch" so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20 \text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{(v_0 \sin 28°)^2}{2g} = 4.4 \text{ m} .$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

29. We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed $|x|$ = 0.055 m as C. Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero

when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270 \text{ N}}{0.02 \text{ m}} = 1.35 \times 10^4 \text{ N/m} .$$

(a) The distance between points A and B is $\vec{F}_g$ and we note that the total sliding distance $\ell + |x|$ is related to the initial height h of the block (measured relative to C) by

$$\frac{h}{\ell + |x|} = \sin \theta$$

where the incline angle θ is 30°. Mechanical energy conservation leads to

$$K_A + U_A = K_C + U_C$$
$$0 + mgh = 0 + \frac{1}{2}kx^2$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})\left(9.8 \text{ m/s}^2\right)} = 0.174 \text{ m} .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30°} = \frac{0.174 \text{ m}}{\sin 30°} = 0.35 \text{ m} .$$

(b) From this result, we find $\ell = 0.35 - 0.055 = 0.29 \text{ m}$, which means that $\Delta y = -\ell \sin \theta = -0.15 \text{ m}$ in sliding from point A to point B. Thus, Eq. 8-18 gives

$$\Delta K + \Delta U = 0$$
$$\frac{1}{2}mv_B^2 + mg\Delta h = 0$$

which yields $v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7 \text{ m/s}$.

33. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10 \text{ N/cm} = 10 \text{ N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields v = 2.8 m/s for m = 0.0038 kg and x = 0.055 m.

(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields v = 2.7 m/s.

37. From Fig. 8-48, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is $K_1 = mv^2/2$ = (0.90 kg)(7.0 m/s)^2/2 = 22 J. The total energy is $E_1 = U_1 + K_1$ = (15 + 22) = 37 J.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. From energy conservation, we have K_2=2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0\ \text{J})}{0.90\ \text{kg}}} = 2.1\ \text{m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\dfrac{35-15}{2-4} = +10\ \text{N}$.

(c) Since the magnitude $F_x > 0$, the force points in the $+x$ direction.

(d) At x = 7.0m, the potential energy is U_3 = 45 J which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x-5), \qquad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x - 5)$, which yields x = 5.7 m.

(e) At x =5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\ \text{J}}{(6-5)\ \text{m}} = -30\ \text{N}$$

The magnitude is $|F_x| = 30\ \text{N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the –x direction.

42. (a) The work is $W = Fd$ = (35 N)(3 m) = 105 J.

(b) The total amount of energy that has gone to thermal forms is (see Eq. 8-31 and Eq. 6-2)

$$\Delta E_{\text{th}} = \mu_k mgd = (0.6)(4\ \text{kg})(9.8\ \text{m/s}^2)(3\ \text{m}) = 70.6\ \text{J}.$$

If 40 J has gone to the block then (70.6 – 40) J = 30.6 J has gone to the floor.

(c) Much of the work (105 J) has been “wasted” due to the 70.6 J of thermal energy generated, but there still remains (105 – 70.6) J = 34.4 J which has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)

43. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1\,\text{J}.$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\,\text{J}.$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x and the y component of Newton's second law are

$$x:\quad F\cos\theta - f = 0$$
$$y:\quad F_N + F\sin\theta - mg = 0,$$

where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives

$$f = F\cos\theta = (7.68)\cos 15.0° = 7.42\ \text{N}$$

and the second gives

$$F_N = mg - F\sin\theta = (3.57)(9.8) - (7.68)\sin 15.0° = 33.0 \text{ N}.$$

Thus

$$\mu_k = \frac{f}{F_N} = \frac{7.42 \text{ N}}{33.0 \text{ N}} = 0.225\,.$$

44. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}\, m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}\,(60)(24^2 - 22^2) + (60)(9.8)(14)\\ &= 1.1\times 10^4 \text{ J}.\end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

45. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25 \text{ kg})\left(9.8 \text{ m/s}^2\right)(12 \text{ m}) = -2.9\times 10^3 \text{ J}\,.$$

(b) The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(25 \text{ kg})(5.6 \text{ m/s})^2 = 3.9\times 10^2 \text{ J}$.

(c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9\times 10^2 \text{ J} - 2.9\times 10^3 \text{ J}}{12 \text{ m}} = 2.1\times 10^2 \text{ N}$$

48. (a) The initial potential energy is

$$U_i = mgy_i = (520 \text{ kg})\left(9.8 \text{ m/s}^2\right)(300 \text{ m}) = 1.53\times 10^6 \text{ J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

(b) Since $f_k = \mu_k F_N = \mu_k mg\cos\theta$ we have $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$ from Eq. 8-31. Now, the hillside surface (of length $d = 500$ m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos\theta = x/d$ where $x = 400$ m. Therefore,

$$\Delta E_{\text{th}} = \mu_k mgd\frac{x}{d} = \mu_k mgx = (0.25)(520)(9.8)(400) = 5.1\times10^5 \text{ J} .$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$\begin{aligned} K_f &= K_i + U_i - U_f - \Delta E_{\text{th}} \\ &= 0 + 1.53\times10^6 - 0 - 5.1\times10^5 \\ &= 0 + 1.02\times10^6\,\text{J}. \end{aligned}$$

(d) From $K_f = \frac{1}{2}mv^2$ we obtain $v = 63$ m/s.

49. We use Eq. 8-31

$$\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.0\,\text{m}) = 50\,\text{J}.$$

and Eq. 7-8

$$W = Fd = (2.0\,\text{N})(5.0\,\text{m}) = 10\,\text{J}.$$

and Eq. 8-31

$$\begin{aligned} W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50 \end{aligned}$$

which yields $\Delta U = -75$ J. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75$ J.

50. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where $h = 1.1$ m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$, where $\mu = 0.60$. Thus, Eq. 8-33 (with $W = 0$) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

where $K_i = \frac{1}{2}mv_i^2$ and $v_i = 6.0$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2\,\text{m}.$$

51. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})(9.8\,\text{m/s}^2)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{\text{max}} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\text{max}}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}.$$

54. We look for the distance along the incline d which is related to the height ascended by $\Delta h = d \sin\theta$. By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $F_N = mg\cos\theta$ which means $f_k = \mu_k mg\cos\theta$. Thus, Eq. 8-33 (with $W = 0$) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{128}{(4.0)(9.8)(\sin 30° + 0.30\cos 30°)} = 4.3\,\text{m}.$$

55. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where $mg = 267$ N) which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times10^2\,\text{J}.$$

(a) The potential energy change is

$$\Delta U = mg(-d\sin\theta) = (267)(-6.1\sin 20°) = -5.6\times10^2\,\text{J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\,\text{N}}{9.8\,\text{m/s}^2}\right)(0.457\,\text{m/s}^2) = 2.8\,\text{J}.$$

Therefore, using Eq. 8-33 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6\times10^2\right) - 1.5\times10^2 = 4.1\times10^2\,\text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$.

57. (a) With x = 0.075 m and $k = 320\,\text{N/m}$, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J. For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $F_N = mg$ which means $f_k = \mu_k F_N = \mu_k mg$. With $d = x$, Eq. 8-31 yields $\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46$ J.

(c) Eq. 8-33 (with $W = 0$) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36\ \text{J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0\ \text{m/s}$.

58. (a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{\text{th}} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with m = (5.29 N)/(9.80 m/s^2)=0.54 kg,

$$h = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(1+0.265/5.29)} = 19.4\ \text{m/s}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2\left(1 - \frac{2f}{w+f}\right) = v_0^2\,\frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} = (20.0\ \text{m/s})\sqrt{\frac{5.29-0.265}{5.29+0.265}} = 19.0\ \text{m/s}\,.$$

65. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length $R = 12$ m that is pulled leftward to an angle θ (corresponding to being at the top of the slide at height $h = 4.0$ m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2\ \text{m/s}$. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R(1-\cos\theta) \Rightarrow \theta = \cos^{-1}\left(1 - \frac{h}{R}\right) = 48°$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore (12)(0.84) = 10 m long.

(b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with $W = 0$):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with m = 25 kg) we obtain f = 49 N.

(c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at θ_1 = 90° measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed v = 6.2 m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R\left(1-\cos\theta_2\right) - R\left(1-\cos\theta_1\right) = -R\cos\theta_2$$

where we have used the fact that cos θ_1 = 0. Thus, with Δh = –4.0 m, we obtain θ_2 =70.5° which means the arc subtends an angle of $|\Delta\theta|$ = 19.5° or 0.34 radians. Multiplying this by the radius gives a slide length of s' = 4.1 m.

(d) We again find the magnitude f' of the frictional force by using Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f' s' \end{aligned}$$

so that we obtain $f' = 1.2 \times 10^2$ N.

89. We note that if the larger mass (block B, m_B = 2 kg) falls d = 0.25 m, then the smaller mass (blocks A, m_A = 1 kg) must increase its height by $h = d\sin 30°$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = m_B g d - m_A g h = 3.7 \text{ J}.$$

105. Since the speed is constant ΔK = 0 and Eq. 8-33 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th(cube)}} + \Delta E_{\text{th(floor)}}.$$

Thus, if W_{applied} = (15)(3.0) = 45 J, and we are told that $\Delta E_{\text{th (cube)}}$ = 20 J, then we conclude that $\Delta E_{\text{th (floor)}}$ = 25 J.

107. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F}\cdot\vec{v} = Fv = \left(110\,\text{N}\right)\left(0.22\,\text{m/s}\right) = 24\,\text{W}.$$

112. (a) The (internal) energy the climber must convert to gravitational potential energy is $\Delta U = mgh = (90)(9.8)(8850) = 7.8 \times 10^6$ J.

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8 \times 10^6 \text{ J}}{1.25 \times 10^6 \text{ J/bar}} \approx 6.2 \text{ bars}.$$

119. (a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2} m \left(v^2 - v_0^2\right).$$

Therefore, $\Delta E = 6.0 \times 10^3$ J.

(b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v / a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W}.$$

(c) and (d) The constant applied force is ma = 30 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

121. We want to convert (at least in theory) the water that falls through h = 500 m into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12}$ m^2 and Δz = 0.75 m. Multiplying this volume by the density ρ = 1000 kg/m^3 leads to

$$m_{\text{total}} = \rho A \Delta z = (1000)\left(8 \times 10^{12}\right)(0.75) = 6 \times 10^{15} \text{kg}$$

for the mass of rainwater. One-third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to 3.2×10^7 s, we obtain

$$P_{\text{avg}} = \frac{\left(2 \times 10^{15}\right)(9.8)(500)}{3.2 \times 10^{7}} = 3.1 \times 10^{11}\,\text{W}.$$

128. Eq. 8-8 leads directly to $\Delta y = \dfrac{68000\ \text{J}}{(9.4\ \text{kg})(9.8\ \text{m/s}^2)} = 738$ m.