Halliday/Resnick/Walker 7e
Chapter 9

1. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and, $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle.

(a) The x coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(2.0\text{ m}) + (8.0\text{ kg})(1.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.1\text{ m}.$$

(b) The y coordinate of the center of mass is

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(1.0\text{ m}) + (8.0\text{ kg})(2.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.3\text{ m}.$$

(c) As the mass of m_3, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where m_3 is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of m_3.

2. We use Eq. 9-5 (with SI units understood).

(a) The x coordinates of the system's center of mass is:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00)(-1.20) + (4.00)(0.600) + (3.00)\,x_3}{2.00 + 4.00 + 3.00} = -0.500$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The y coordinates of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00)(0.500) + (4.00)(-0.750) + (3.00)\,y_3}{2.00 + 4.00 + 3.00} = -0.700\,.$$

Solving the equation yields $y_3 = -1.43$ m.

4. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We'll refer to the large 35 cm × 10 cm piece (shown to the left of the y axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at $(x_1, y_1) = (-5.0\text{ cm}, -2.5\text{ cm})$. The top 20 cm × 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its

center of mass is at $(x_2,y_2) = (10$ cm, 12.5 cm). The bottom 10 cm x 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at $(x_3,y_3) = (5$ cm, -15 cm).

(a) The x coordinate of the center of mass for the plate is

$$x_{\text{com}} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm}.$$

(b) The y coordinate of the center of mass for the plate is

$$y_{\text{com}} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm}.$$

5. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{\text{com}} = 0$.

(b) To find y_{com}, we note that $3m_H y_{\text{com}} = m_N(y_N - y_{\text{com}})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{\left(10.14\times10^{-11}\text{ m}\right)^2 - \left(9.4\times10^{-11}\text{ m}\right)^2} = 3.803\times10^{-11}\text{ m}.$$

Thus,

$$y_{\text{com}} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(14.0067)\left(3.803\times10^{-11}\text{ m}\right)}{14.0067 + 3(1.00797)} = 3.13\times10^{-11}\text{ m}$$

where Appendix F has been used to find the masses.

9. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$.

(a) The location of the first stone (of mass m_1) at $t = 300 \times 10^{-3}$ s is

$$y_1 = (1/2)gt^2 = (1/2)(9.8)(300 \times 10^{-3})^2 = 0.44 \text{ m},$$

and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_2 = (1/2)gt^2 = (1/2)(9.8)(300 \times 10^{-3} - 100 \times 10^{-3})^2 = 0.20 \text{ m}.$$

Thus, the center of mass is at

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44\text{ m}) + 2m_1(0.20\text{ m})}{m_1 + 2m_2} = 0.28\text{ m}.$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is

$$v_2 = g(t - 100 \times 10^{-3}\ \text{s}).$$

Thus, the center-of-mass speed at $t = 300 \times 10^{-3}$ s is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1 (9.8)\left(300 \times 10^{-3}\right) + 2m_1 (9.8)\left(300 \times 10^{-3} - 100 \times 10^{-3}\right)}{m_1 + 2m_1}$$
$$= 2.3\ \text{m/s}.$$

11. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$. At t = 3.0 s, the location of the automobile (of mass m_1) is $x_1 = \frac{1}{2}at^2 = \frac{1}{2}\left(4.0\ \text{m/s}^2\right)(3.0\ \text{s})^2 = 18$ m, while that of the truck (of mass m_2) is $x_2 = vt$ = (8.0 m/s)(3.0s) = 24 m. The speed of the automobile then is $v_1 = at = \left(4.0\ \text{m/s}^2\right)\left(3.0\ \text{s}\right) = 12$ m/s, while the speed of the truck remains v_2 = 8.0 m/s.

(a) The location of their center of mass is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000\ \text{kg})(18\ \text{m}) + (2000\ \text{kg})(24\ \text{m})}{1000\ \text{kg} + 2000\ \text{kg}} = 22\ \text{m}.$$

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000\ \text{kg})(12\ \text{m/s}) + (2000\ \text{kg})(8.0\ \text{m/s})}{1000\ \text{kg} + 2000\ \text{kg}} = 9.3\ \text{m/s}.$$

13. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block1, this acceleration is to the right (the $\hat{\text{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\text{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{\text{com}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\text{i}}) + (0.4)(-0.4g\hat{\text{j}})}{0.6 + 0.4} = (2.35\ \hat{\text{i}} - 1.57\ \hat{\text{j}})\ \text{m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{\text{v}}_{\text{com}} = (2.35\ \hat{\text{i}} - 1.57\hat{\text{j}})\ t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the y-component to the x-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Eq. 3-6 leads to $\theta = -34^\circ$. The path of the center of mass is therefore straight, at downward angle 34°.

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0\,y} - gt$ and this is zero at time $t = v_{0\,y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60^\circ\cos 60^\circ = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60^\circ = 15.3\text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60^\circ = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m}.$$

19. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100\ \text{kg})\left((51\ \text{km/h})^2 - (41\ \text{km/h})^2\right)\\ &= 9.66\times10^4\ \text{kg}\cdot(\text{km/h})^2\left((10^3\ \text{m/km})(1\ \text{h}/3600\ \text{s})\right)^2\\ &= 7.5\times10^4\ \text{J}.\end{aligned}$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41\ \text{km/h})^2 + (51\ \text{km/h})^2} = 65.4\ \text{km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100\ \text{kg})(65.4\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right) = 3.8\times10^4\ \text{kg}\cdot\text{m/s}.$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\ \text{km/h}}{51\ \text{km/h}}\right) = 39°.$$

21. We infer from the graph that the horizontal component of momentum p_x is 4.0 kg·m/s. Also, its initial magnitude of momentum p_o is 6.0 kg·m/s. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad\Rightarrow\quad \theta_o = 48° .$$

23. The initial direction of motion is in the +x direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4\ \text{N}\cdot\text{s}}{2.70\times10^{-2}\ \text{s}} = 1.20\times10^3\ \text{N}$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{avg}\Delta t = mv_f - mv_i.$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. Thus,

$$v_f = \frac{mv_i - F_{avg}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times10^{-3}\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

(a) The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20 \times 10^3$ N.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

24. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law: $F - mg = ma$, where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation: $v = \sqrt{2gh}$, where $h = 12$ m, and since the deceleration a reduces the speed to zero over a distance $d = 0.30$ m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 9-29,

$$\int F dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction) which yields (70) $\sqrt{2(9.8)(12)} = 1.1 \times 10^3 \text{kg} \cdot \text{m/s}$. Expressing this as a range (in kN·s) we estimate

$$1.0 < \int F\,dt < 1.2.$$

25. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration $a_{\max}$ of the paratrooper (of mass m and initial speed v = 56 m/s) is found from Newton's second law

$$F_{\text{snow}} - mg = ma_{\max}$$

where we require $F_{\text{snow}} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\max}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{\max}} = \frac{mv^2}{2\left(F_{\text{snow}} - mg\right)} \approx \frac{(85\,\text{kg})(56\,\text{m/s})^2}{2\left(1.2\times10^5\,\text{N}\right)} = 1.1 \text{ m.}$$

(b) His short trip through the snow involves a change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s}) = -4.8\times10^3\,\text{kg}\cdot\text{m/s},$$

or $|\Delta\vec{p}| = 4.8\times10^3\,\text{kg}\cdot\text{m/s}$. The negative value of the initial velocity is due to the fact that downward is the negative direction. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

27. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)\left(3\times10^{-3}\,\text{kg}\right)(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N.}$$

32. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the t converted to seconds). With m = 0.058 kg and v = 34 m/s, we apply the impulse-momentum theorem:

$$\begin{aligned}\int F_{\text{wall}}\,dt = m\vec{v}_f - m\vec{v}_i \quad &\Rightarrow \quad \int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt = m(+v) - m(-v) \\ &\Rightarrow \quad \frac{1}{2}F_{\max}(0.002\,\text{s}) + F_{\max}(0.002\,\text{s}) + \frac{1}{2}F_{\max}(0.002\,\text{s}) = 2mv\end{aligned}$$

which yields $F_{\max}(0.004\,\text{s}) = 2(0.058\,\text{kg})(34\,\text{m/s}) = 9.9 \times 10^2$ N.

35. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then

$$m_s v_s + m_m v_m = 0 \rightarrow \; v_m = -m_s v_s / m_m.$$

We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068 \text{ kg})(4.0 \text{ m/s})}{91 \text{ kg}} = -3.0\times10^{-3} \text{ m/s},$$

or $|v_m| = 3.0\times10^{-3}$ m/s. The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

36. We apply Eq. 9-17, with $M = \sum m = 1.3$ kg,

$$\begin{aligned} M\vec{v}_{\text{com}} &= m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C \\ (1.3)\,\left(-0.40\,\hat{\text{i}}\right) &= (0.50)\,\vec{v}_A + (0.60)\left(0.20\,\hat{\text{i}}\right) + (0.20)\left(0.30\,\hat{\text{i}}\right) \end{aligned}$$

which leads to $\vec{v}_A = -1.4\,\hat{\text{i}}$ in SI units (m/s).

37. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4\times10^3 \text{ km/h}.$$

43. Our notation is as follows: the mass of the original body is M = 20.0 kg; its initial velocity is $\vec{v}_0 = 200\hat{\text{i}}$ in SI units (m/s); the mass of one fragment is m_1 = 10.0 kg; its velocity is $\vec{v}_1 = -100\hat{\text{j}}$ in SI units; the mass of the second fragment is m_2 = 4.0 kg; its velocity is $\vec{v}_2 = -500\hat{\text{i}}$ in SI units; and, the mass of the third fragment is m_3 = 6.00 kg.

(a) Conservation of linear momentum requires $M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$, which (using the above information) leads to

$$\vec{v}_3 = (1.00\times10^3\,\hat{\mathrm{i}} - 0.167\times10^3\,\hat{\mathrm{j}})\ \mathrm{m/s}$$

in SI units. The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{1000^2 + (-167)^2} = 1.01\times10^3\ \mathrm{m/s}$. It points at $\tan^{-1}$ (–167/1000) = –9.48° (that is, at 9.5° measured clockwise from the +x axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23\times10^6\ \mathrm{J}.$$

45. (a) We choose +x along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2\ \mathrm{g})(672\ \mathrm{m/s}) = (5.2\ \mathrm{g})(428\ \mathrm{m/s}) + (700\ \mathrm{g})\vec{v}_2$$

which yields v_2 = 1.81 m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2\ \mathrm{g})(672\ \mathrm{m/s})}{5.2\ \mathrm{g} + 700\ \mathrm{g}} = 4.96\ \mathrm{m/s}.$$

46. We refer to the discussion in the textbook (see Sample Problem 9-8, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1\times10^2\ \mathrm{m/s}.$$

48. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \Rightarrow v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since v_0 = 0 (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2\mu_k gd_A} = \sqrt{2(0.13)(9.8)(8.2)} = 4.6\ \mathrm{m/s}.$$

(b) Similarly, $v_B = \sqrt{2\mu_k gd_B} = \sqrt{2(0.13)(9.8)(6.1)} = 3.9\ \mathrm{m/s}.$

(c) Let the speed of car *B* be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6)+(1400)(3.9)}{1400} = 7.5 \text{ m/s}.$$

(d) The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location – that the cars do not slide appreciably during Δt – which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

50. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\,\text{kg})(1000\,\text{m/s}) = (5.0\,\text{kg})\vec{v} + (0.01\,\text{kg})(400\,\text{m/s})$$

which yields $\vec{v} = 1.2\,\text{m/s}$. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\,\text{kg})(1.2\,\text{m/s})^2 = (5.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)h$$

which gives the result $h = 0.073$ m.

51. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses $m_1 = 5$ kg and $m_2 = 10$ kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \quad \Rightarrow \quad (5)(3)+(10)(2) = 5\vec{v}_{1f} + (10)(2.5)$$

which yields $\vec{v}_{1f} = 2$. Thus, the speed of the 5 kg block immediately after the collision is $2.0\,\text{m/s}$.

(b) We find the reduction in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5)(3)^2 + \frac{1}{2}(10)(2)^2 - \frac{1}{2}(5)(2)^2 - \frac{1}{2}(10)(2.5)^2 = -1.25\ \text{J} \approx -1.3\ \text{J}.$$

(c) In this new scenario where $\vec{v}_{2f} = 4.0\,\text{m/s}$, momentum conservation leads to $\vec{v}_{1f} = -1.0\,\text{m/s}$ and we obtain $\Delta K = +40\ \text{J}$.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

52. The total momentum immediately before the collision (with $+x$ upward) is

$$p_i = (3.0\ \text{kg})(20\ \text{m/s}) + (2.0\ \text{kg})(-12\ \text{m/s}) = 36\ \text{kg·m/s}.$$

Their momentum immediately after, when they constitute a combined mass of $M = 5.0$ kg, is $p_f = (5.0\ \text{kg})\vec{v}$. By conservation of momentum, then, we obtain $\vec{v} = 7.2$ m/s, which becomes their "initial" velocity for their subsequent free-fall motion. We can use Ch. 2 methods or energy methods to analyze this subsequent motion; we choose the latter. The level of their collision provides the reference ($y = 0$) position for the gravitational potential energy, and we obtain

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}Mv_0^2 + 0 = 0 + Mgy_{\text{max}}.$$

Thus, with $v_0 = 7.2$ m/s, we find $y_{\text{max}} = 2.6$ m.

55. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}.$$

Using SI units (so $m_1 = 0.34$ kg), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2 - 0.66}{1.2 + 0.66}\right)(0.34) = 0.099\ \text{kg}.$$

(b) The velocity of the second cart is given by Eq. 9-68:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34)}{0.34 + 0.099}\right)(1.2) = 1.9\ \text{m/s}.$$

(c) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93 \ \text{m/s}.$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

56. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6)(5.5) + (2.4)(2.5) - (2.4)(4.9)}{1.6} = 1.9 \ \text{m/s}.$$

(b) The block continues going to the right after the collision.

(c) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}(1.6)(5.5)^2 + \frac{1}{2}(2.4)(2.5)^2 = 31.7 \ \text{J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2}(1.6)(1.9)^2 + \frac{1}{2}(2.4)(4.9)^2 = 31.7 \ \text{J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

58. We use Eq 9-67 and 9-68 to find the velocities of the particles after their first collision (at x = 0 and t = 0):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-0.1 \text{ kg}}{0.7 \text{ kg}} (2.0 \text{ m/s}) = \frac{-2}{7} \text{ m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{0.6 \text{ kg}}{0.7 \text{ kg}} (2.0 \text{ m/s}) = \frac{12}{7} \text{ m/s} \approx 1.7 \text{ m/s}.$$

At a rate of motion of 1.7 m/s, $2x_w$ = 140 cm (the distance to the wall and back to x= 0) will be traversed by particle 2 in 0.82 s. At t = 0.82 s, particle 1 is located at

$$x = (-2/7)(0.82) = -23 \text{ cm},$$

and particle 2 is "gaining" at a rate of (10/7) m/s leftward; this is their relative velocity at that time. Thus, this "gap" of 23 cm between them will be closed after an additional time of (0.23 m)/(10/7 m/s) = 0.16 s has passed. At this time (t = 0.82 + 0.16 = 0.98 s) the two particles are at x = (–2/7)(0.98) = –28 cm.

59. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 9-67,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \ .$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 \ .$$

We combine this with $v_{1f} = v_{1i}/4$ to obtain $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2 \text{ kg}$.

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s}.$$

60. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 g h = \frac{1}{2} m_1 v^2 \ \Rightarrow \ v = \sqrt{2gh} = 3.7 \ \text{m/s}.$$

(a) We now treat the elastic collision (with SI units) using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 - 2.5}{0.5 + 2.5}(3.7) = -2.47$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5)}{0.5 + 2.5}(3.7) = 1.23 \ \text{m/s}.$$

75. (a) We use Eq. 9-68 twice:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.5m_1}(4.00 \text{ m/s}) = \frac{16}{3} \text{ m/s}$$

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \frac{2m_2}{1.5m_2}(16/3 \text{ m/s}) = \frac{64}{9} \text{ m/s} = 7.11 \text{ m/s} .$$

(b) Clearly, the speed of block 3 is greater than the (initial) speed of block 1.

(c) The kinetic energy of block 3 is

$$K_{3f} = \tfrac{1}{2} m_3 v_3^{\,2} = \left(\tfrac{1}{2}\right)^3 m_1 \left(\tfrac{16}{9}\right)^2 v_{1i}^{\,2} = \frac{64}{81} K_{1i} .$$

We see the kinetic energy of block 3 is less than the (initial) K of block 1. In the final situation, the initial K is being shared among the three blocks (which are all in motion), so this is not a surprising conclusion.

(d) The momentum of block 3 is

$$p_{3f} = m_3 v_3 = \left(\tfrac{1}{2}\right)^2 m_1 \left(\tfrac{16}{9}\right) v_{1i} = \frac{4}{9} p_{1i}$$

and is therefore less than the initial momentum (both of these being considered in magnitude, so questions about ± sign do not enter the discussion).

79. We convert mass rate to SI units: R = 540/60 = 9.00 kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-87:

$$R\, v_{\text{rel}} = M\,|a|$$

so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\, v_{\text{rel}}$ (so as to cancel that effect).

$$F = Rv_{\text{rel}} = (9.00)(3.20) = 28.8 \,\text{N} .$$

89. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M + m} = \frac{m(v-u) - Mu}{M + m} = 0.$$

This yields

$$u = \frac{mv}{M+m} = \frac{(80\text{ kg})(2.5\text{ m/s})}{320\text{ kg} + 80\text{ kg}} = 0.50\text{ m/s}.$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

110. (a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\vec{p}_{n\,i} = \vec{p}_e + \vec{p}_v + \vec{p}_{n\,r} \quad \Rightarrow \quad 0 = (-1.2\times10^{-22})\,\hat{\mathrm{i}} + (-6.4\times10^{-23})\,\hat{\mathrm{j}} + \vec{p}_{n\,r}$$

Thus, $\vec{p}_{n\,r} = (1.2\times10^{-22}\,\text{kg}\cdot\text{m/s})\,\hat{\mathrm{i}} + (6.4\times10^{-23}\,\text{kg}\cdot\text{m/s})\,\hat{\mathrm{j}}$. Its magnitude is

$$|\vec{p}_{n\,r}| = \sqrt{\left(1.2\times10^{-22}\right)^2 + \left(6.4\times10^{-23}\right)^2} = 1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}.$$

(b) The angle measured from the $+x$ axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}}{1.2\times10^{-22}}\right) = 28°.$$

(c) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\right)^2}{2\left(5.8\times10^{-26}\right)} = 1.6\times10^{-19}\ \text{J}.$$

119. (a) The magnitude of the impulse is equal to the change in momentum:

$$J = mv - m(-v) = 2mv = 2(0.140\text{ kg})(7.80\text{ m/s}) = 2.18\text{ kg}\cdot\text{m/s}$$

(b) Since in the calculus sense the average of a function is the integral of it divided by the corresponding interval, then the average force is the impulse divided by the time Δt. Thus, our result for the magnitude of the average force is $2mv/\Delta t$. With the given values, we obtain

$$F_{\text{avg}} = \frac{2(0.140\text{ kg})(7.80\text{ m/s})}{0.00380\text{ s}} = 575\text{ N}.$$