Halliday/Resnick/Walker 7e
Chapter 13 – Gravitation

1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.2\,\mathrm{kg}\right)\left(2.4\,\mathrm{kg}\right)}{2.3\times10^{-12}\,\mathrm{N}}} = 19\,\mathrm{m}$$

2. We use subscripts s, e, and m for the Sun, Earth and Moon, respectively.

$$\frac{F_{sm}}{F_{em}} = \frac{\dfrac{Gm_sm_m}{r_{sm}^2}}{\dfrac{Gm_em_m}{r_{em}^2}} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2$$

Plugging in the numerical values (say, from Appendix C) we find

$$\frac{1.99\times10^{30}}{5.98\times10^{24}}\left(\frac{3.82\times10^{8}}{1.50\times10^{11}}\right)^2 = 2.16.$$

3. The gravitational force between the two parts is

$$F = \frac{Gm\left(M-m\right)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to m and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}\left(M-2m\right) \Rightarrow M = 2m$$

which leads to the result m/M= 1/2.

4. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\mathrm{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8}\ \angle\ 60.6^\circ\right).$$

(a) The magnitude of the force is 2.13×10^{-8} N.

(b) The direction of the force relative to the +x axis is $60.6°$.

5. At the point where the forces balance $GM_e m / r_1^2 = GM_s m / r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{\left(d - r_1\right)^2}.$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s} + \sqrt{M_e}} = \frac{\left(150\times10^9 \text{ m}\right)\sqrt{5.98\times10^{24} \text{ kg}}}{\sqrt{1.99\times10^{30} \text{ kg}} + \sqrt{5.98\times10^{24} \text{ kg}}} = 2.60\times10^8 \text{ m.}$$

Values for M_e, M_s, and d can be found in Appendix C.

7. We require the magnitude of force (given by Eq. 13-1) exerted by particle C on A be equal to that exerted by B on A. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2} .$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "m_A") solve for r. We find $r = 5d$. Thus, particle C is placed on the x axis, to left of particle A (so it is at a negative value of x), at $x = -5.00d$.

14. We follow the method shown in Sample Problem 13-3. Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

But since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600 \text{ N}\right)\frac{1.61\times10^3 \text{ m}}{6.37\times10^6 \text{ m}} = -0.303 \text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

15. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\,\text{kg}\right)}{\left(4.9\,\text{m}/\text{s}^2\right)}} - 6.37\times10^6\,\text{m} = 2.6\times10^6\,\text{m}.$$

25. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_g M = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2\left(9.8\ \text{m/s}^2\right) = 3.8\ \text{m/s}^2.$$

(c) If v is the escape speed, then, for a particle of mass m

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad\Rightarrow\quad v = \sqrt{\frac{2GM}{R}}.$$

For Mars

$$v = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(0.11\right)\left(5.98\times10^{24}\ \text{kg}\right)}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

30. (a) From Eq. 13-28, we see that $v_o = \sqrt{\frac{GM}{2R_E}}$ in this problem. Using energy conservation, we have

$$\frac{1}{2}mv_o^2 - GMm/R_E = -GMm/r$$

which yields $r = 4R_E/3$. So the multiple of R_E is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of R_E is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

33. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m})} = 1.7 \times 10^3 \text{ m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)}.$$

The solution for h is

$$\begin{aligned} h &= \frac{2a_gR^2}{2a_gR - v^2} - R = \frac{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m})^2}{2(3.0 \text{ m/s}^2)(500 \times 10^3 \text{ m}) - (1000 \text{ m/s})^2} - (500 \times 10^3 \text{ m}) \\ &= 2.5 \times 10^5 \text{ m}. \end{aligned}$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for v is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\text{ m/s}^2)(500\times10^3\text{ m}) - \frac{2(3.0\text{ m/s}^2)(500\times10^3\text{ m})^2}{(500\times10^3\text{ m})+(1000\times10^3\text{ m})}}$$
$$= 1.4\times10^3\text{ m/s}.$$

83. (a) We note that *height* = $R - R_{\text{Earth}}$ where $R_{\text{Earth}} = 6.37 \times 10^6$ m. With $M = 5.98 \times 10^{24}$ kg, $R_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2}m\,(3.70\times10^3)^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2}m\,(3.70\times10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km ≈ 1.03×10^3 km above the Earth's surface.

84. Energy conservation for this situation may be expressed as follows:

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} &= \frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \end{aligned}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$ and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \Rightarrow v_2 = \sqrt{\frac{2GM}{R}} = 2.4\times10^4\text{ m/s}.$$