Halliday/Resnick/Walker 7e
Chapter 15 - Oscillations

1. (a) The amplitude is half the range of the displacement, or $x_m = 1.0$ mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi\left(120\ \text{Hz}\right)\left(1.0\times10^{-3}\ \text{m}\right) = 0.75\ \text{m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = \left(2\pi f\right)^2 x_m = \left(2\pi\left(120\ \text{Hz}\right)\right)^2\left(1.0\times10^{-3}\ \text{m}\right) = 5.7\times10^2\ \text{m/s}^2.$$

5. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\ \text{s}) = 2.00$ Hz.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00\ \text{Hz}) = 12.6$ rad/s.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k: $k = m\omega^2 = (0.500\ \text{kg})(12.6\ \text{rad/s})^2 = 79.0$ N/m.

(e) Let x_m be the amplitude. The maximum speed is $v_m = \omega x_m = (12.6\ \text{rad/s})(0.350\ \text{m}) = 4.40$ m/s.

(f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0\ \text{N/m})(0.350\ \text{m}) = 27.6$ N.

7. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$ must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

8. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 15-12 leads to

$$\omega = \sqrt{\frac{k}{m_{\text{car}}/4}} \quad \Rightarrow \quad k = \frac{1}{4}(1450\text{kg})(6\pi \text{ rad/s})^2 = 1.29\times10^5\,\text{N/m}.$$

(b) If the new mass being supported by the four springs is m_{total} = [1450 + 5(73)] kg = 1815 kg, then Eq. 15-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{m_{\text{total}}/4}} \quad \Rightarrow f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29\times10^5 \text{ N/m}}{(1815/4)\text{ kg}}} = 2.68\text{Hz}.$$

13. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\text{ m/s}^2}{1.0\times10^{-6}\text{m}}} = 498\text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

14. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m = \frac{1}{2}d$ and ω = 0.503 rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$.

$$\begin{aligned} x &= x_m\cos(\omega t + \phi) \\ \frac{1}{4}d &= \left(\frac{1}{2}d\right)\cos(0.503t + 0) \\ \frac{1}{2} &= \cos(0.503t) \end{aligned}$$

which has t = 2.08 h as the smallest positive root. The calculator is in radians mode during this calculation.

15. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is

given by $F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\ \text{m/s}^2)}{(2\pi \times 2.0\ \text{Hz})^2} = 0.031\ \text{m}.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

17. (a) Eq. 15-8 leads to

$$a = -\omega^2 x \Rightarrow \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123}{0.100}}$$

which yields $\omega = 35.07$ rad/s. Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Eq. 15-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad m = \frac{400\ \text{N/m}}{(35.07\ \text{rad/s})^2} = 0.325\text{kg}.$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow x_m = \frac{m}{k}v^2 + x^2.$$

Consequently, $x_m = \sqrt{(0.325/400)(13.6)^2 + (0.100)^2} = 0.400\text{m}.$

25. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\max} = \mu_s\, mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M}x_m = \mu_s g$$

which leads to $x_m = 0.22$ m.

31. The total energy is given by $E = \frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

(a) The fraction of the energy that is kinetic is

$$\frac{K}{E} = \frac{E-U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

where the result from part (b) has been used.

(b) When $x = \frac{1}{2}x_m$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kx_m^2$. The ratio is

$$\frac{U}{E} = \frac{\frac{1}{8}kx_m^2}{\frac{1}{2}kx_m^2} = \frac{1}{4} = 0.25.$$

(c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve $x^2/x_m^2 = 1/2$ for x. We should get $x = x_m/\sqrt{2}$.

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently x_m = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant k:

$$\frac{1}{2}k\,x_m^2 = 6.0 \text{ J} \quad \Rightarrow \quad k = 8.3 \times 10^2 \text{ N/m}.$$

33. (a) Eq. 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000 \text{ N/m}}{5.00 \text{ kg}}} = 2.25 \text{ Hz}.$$

(b) With x_o = 0.500 m, we have $U_0 = \frac{1}{2}kx_0^2 = 125$ J.

(c) With v_o = 10.0 m/s, the initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2 = 250$ J.

(d) Since the total energy $E = K_o + U_o$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = 0.866 \text{ m}.$$

36. We note that the spring constant is $k = 4\pi^2 m_1/T^2 = 1.97 \times 10^5$ N/m. It is important to determine where in its simple harmonic motion (which "phase" of its motion) block 2 is when

the impact occurs. Since $\omega = 2\pi/T$ and the given value of t (when the collision takes place) is one-fourth of T, then $\omega t = \pi/2$ and the location then of block 2 is $x = x_m\cos(\omega t + \phi)$ where $\phi = \pi/2$ which gives $x = x_m\cos(\pi/2 + \pi/2) = -x_m$. This means block 2 is at a turning point in its motion (and thus has zero speed right before the impact occurs); this means, too, that the spring is stretched an amount of 1 cm = 0.01 m at this moment. To calculate its after-collision speed (which will be the same as that of block 1 right after the impact, since they stick together in the process) we use momentum conservation and obtain (4.0 kg)(6.0 m/s)/(6.0 kg) = 4.0 m/s. Thus, at the end of the impact itself (while block 1 is still at the same position as before the impact) the system (consisting now of a total mass $M = 6.0$ kg) has kinetic energy $\frac{1}{2}(6.0\text{ kg})(4.0\text{ m/s})^2 = 48$ J and potential energy $\frac{1}{2}(1.97 \times 10^5\text{ N/m})(0.010\text{ m})^2 \approx 10$ J, meaning the total mechanical energy in the system at this stage is approximately 58 J. When the system reaches its new turning point (at the new amplitude X) then this amount must equal its (maximum) potential energy there: $\frac{1}{2}(1.97 \times 10^5)\, X^2$. Therefore, we find

$$X = \sqrt{2(58)/(1.97 \text{ x } 10^5)} = 0.024 \text{ m}.$$

42. (a) Comparing the given expression to Eq. 15-3 (after changing notation $x \rightarrow \theta$), we see that $\omega = 4.43$ rad/s. Since $\omega = \sqrt{g/L}$ then we can solve for the length: $L = 0.499$ m.

(b) Since $v_m = \omega x_m = \omega L\theta_m = (4.43\text{ rad/s})(0.499\text{ m})(0.0800\text{ rad})$ and $m = 0.0600$ kg, then we can find the maximum kinetic energy: $\frac{1}{2}mv_m^2 = 9.40 \times 10^{-4}$ J.

53. Replacing x and v in Eq. 15-3 and Eq. 15-6 with θ and $d\theta/dt$, respectively, we identify 4.44 rad/s as the angular frequency ω. Then we evaluate the expressions at $t = 0$ and divide the second by the first:

$$\left(\frac{d\theta/dt}{\theta}\right)_{\text{at } t = 0} = -\,\omega \tan\phi\,.$$

(a) The value of θ at $t = 0$ is 0.0400 rad, and the value of $d\theta/dt$ then is –0.200 rad/s, so we are able to solve for the phase constant: $\phi = \tan^{-1}[0.200/(0.0400 \text{ x } 4.44)] = 0.845$ rad.

(b) Once ϕ is determined we can plug back in to $\theta_o = \theta_m\cos\phi$ to solve for the angular amplitude. We find $\theta_m = 0.0602$ rad.

58. Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m}.$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6.0 %.

65. (a) From the graph, we find x_m = 7.0 cm = 0.070 m, and T = 40 ms = 0.040 s. Thus, the angular frequency is $\omega = 2\pi/T$ = 157 rad/s. Using m = 0.020 kg, the maximum kinetic energy is then $\frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 x_m^2$ = 1.2 J.

(b) Using Eq. 15-5, we have $f = \omega/2\pi$ = 50 oscillations per second. Of course, Eq. 15-2 can also be used for this.

75. (a) Hooke's law readily yields k = (15 kg)(9.8 m/s^2)/(0.12 m) = 1225 N/m. Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

(b) We are told f = 2.00 Hz = 2.00 cycles/sec. Since a cycle is equivalent to 2π radians, we have $\omega = 2\pi(2.00) = 4\pi$ rad/s (understood to be valid to three significant figures). Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \;\Rightarrow\; m = \frac{1225\text{ N/m}}{(4\pi\text{ rad/s})^2} = 7.76\text{kg}.$$

Consequently, the weight of the package is mg = 76.0 N.

78. (a) The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = (2\pi(1000\text{ Hz}))^2(0.00040\text{ m}) = 1.6\times10^4\text{ m/s}^2.$$

(b) Similarly, in the discussion after Eq. 15-6, we find $v_m = \omega x_m$ so that

$$v_m = (2\pi(1000\text{ Hz}))(0.00040\text{ m}) = 2.5\text{ m/s}.$$

(c) From Eq. 15-8, we have (in absolute value)

$$|a| = (2\pi(1000\text{ Hz}))^2(0.00020\text{ m}) = 7.9\times10^3\text{ m/s}^2.$$

(d) This can be approached with the energy methods of §15-4, but here we will use trigonometric relations along with Eq. 15-3 and Eq. 15-6. Thus, allowing for both roots stemming from the square root,

$$\sin(\omega t+\phi)=\pm\sqrt{1-\cos^2(\omega t+\phi)} \quad\Rightarrow\quad -\frac{v}{\omega x_m}=\pm\sqrt{1-\frac{x^2}{x_m^2}}.$$

Taking absolute values and simplifying, we obtain

$$|v|=2\pi f\sqrt{x_m^2-x^2}=2\pi(1000)\sqrt{0.00040^2-0.00020^2}=2.2\text{ m/s}.$$

86. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{\text{eff}}=\sqrt{g^2+\left(\frac{v^2}{R}\right)^2}.$$

Then, since frequency is the reciprocal of the period, Eq. 15-28 leads to

$$f=\frac{1}{2\pi}\sqrt{\frac{g_{eff}}{L}}=\frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2+v^4/R^2}}{L}}.$$

With v = 70 m/s, R = 50m, and L = 0.20 m, we have $f=3.53\text{ s}^{-1}=3.53\text{ Hz}$.

89. (a) Hooke's law readily yields (0.300 kg)(9.8 m/s^2)/(0.0200 m) = 147 N/m.

(b) With m = 2.00 kg, the period is

$$T=2\pi\sqrt{\frac{m}{k}}=0.733\text{ s}.$$

100. (a) Eq. 15-21 leads to

$$E=\frac{1}{2}kx_m^2\Rightarrow x_m=\sqrt{\frac{2E}{k}}=\sqrt{\frac{2(4.0)}{200}}=0.20\text{ m}.$$

(b) Since $T=2\pi\sqrt{m/k}=2\pi\sqrt{0.80/200}\approx 0.4\text{ s}$, then the block completes 10/0.4 = 25 cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \Rightarrow 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

Therefore, when $x = 0.15$ m, we find $v = 2.1$ m/s.

102. The period formula, Eq. 15-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From Figure 15-59, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M\left(L + R\right)^2,$$

where $M = 1.0\text{ kg}$. Thus, Eq. 15-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M\left(L+R\right)^2}{Mg\left(L+R\right)}}$$

which leads to $L = 0.8315$ m.

106. $m = \dfrac{0.108\text{ kg}}{6.02 \times 10^{23}} = 1.8 \times 10^{-25}$ kg. Using Eq. 15-12 and the fact that $f = \omega/2\pi$, we have

$$1\times10^{13}\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi\times10^{13}\right)^2\left(1.8\times10^{-25}\right) \approx 7\times10^2\text{ N/m.}$$