Halliday/Resnick/Walker 7e
Chapter 16 – Waves 1

1. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T = 4(0.170 \text{ s}) = 0.680$ s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

2. (a) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}.$$

(b) The speed of the wave is

$$v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}.$$

3. Let $y_1 = 2.0$ mm (corresponding to time t_1) and $y_2 = -2.0$ mm (corresponding to time t_2). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0) \;.$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2 = 0.011$ s (or 1.1 ms).

5. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700\text{ m} = 700\text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

6. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\max} = 2\pi f y_m = \left(4.0\,\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

8. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t) \quad .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2} \quad .$$

Therefore, where $y = 12$, u must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

9. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, i.e., $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t$ = 15 m/s, where d = 0.060 m and t = 0.0040 s. The angular wave number is $k = 2\pi/\lambda$ where λ = 0.40 m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = k\,v = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right [in the $+x$ direction] – see section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times10^2\, t) .$$

12. The volume of a cylinder of height ℓ is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho\pi d^2\, \ell/4}{\ell} = \frac{\pi\rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2 .$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

13. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope: $\mu = m/L$ = (0.0600 kg)/(2.00 m) = 0.0300 kg/m. Thus

$$v=\sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}}=129\,\text{m/s}.$$

14. From $v=\sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}}=\frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}}=\sqrt{2}.$$

15. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\text{ m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\text{ rad/s})/(2.0\text{ m}^{-1}) = 15\text{ m/s}.$$

(b) Since the wave speed is given by $v=\sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau=\mu v^2=\left(1.6\times10^{-4}\,\text{kg/m}\right)\left(15\,\text{m/s}\right)^2=0.036\,\text{N}.$$

16. We use $v=\sqrt{\tau/\mu}\propto\sqrt{\tau}$ to obtain

$$\tau_2=\tau_1\left(\frac{v_2}{v_1}\right)^2=\left(120\,\text{N}\right)\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2=135\text{N}.$$

17. (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wave speed is given by $v=\sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}\,/f$ and the angular wave number is

$$k=\frac{2\pi}{\lambda}=2\pi f\sqrt{\frac{\mu}{\tau}}=2\pi\left(100\,\text{Hz}\right)\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}}=141\,\text{m}^{-1}.$$

(c) The frequency is $f = 100$ Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

19. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55\text{ cm} - 15\text{ cm}) = 40\text{ cm} = 0.40\text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12\text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12\text{ m/s})/(0.40\text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30\text{ Hz}) = 0.033\text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30\text{ Hz})\,(5.0\text{ cm}) = 940\text{ cm/s} = 9.4\text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\text{ m}) = 16\text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30\text{ Hz}) = 1.9\times10^2$ rad/s

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is 4.0×10^{-2} m. The formula for the displacement gives $y(0, 0) = y_m \sin\phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}$. The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. Using the results obtained above, the expression for the displacement is

$$y(x,t) = \left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

24. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

25. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\ \text{ms}}$ = 500 Hz, and that the wavelength λ = 0.20 m. We also note from the graph that the maximum value of dK/dt is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\tfrac{1}{2}\,\mu\, v\, \omega^2 y_m^{\ 2} = 10$$

with SI units understood. Substituting in μ = 0.002 kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2\mu\lambda\ f^3}} = 0.0032\ \text{m}\ .$$

29. The displacement of the string is given by

$$y = y_m \sin(kx-\omega t) + y_m \sin(kx-\omega t+\phi) = 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx-\omega t+\tfrac{1}{2}\phi\right),$$

where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41y_m\ .$$

32. (a) We use Eq. 16-26 and Eq. 16-33 with μ = 0.00200 kg/m and y_m = 0.00300 m. These give $v = \sqrt{\tau/\mu}$ = 775 m/s and

$$P_{\text{avg}} = \tfrac{1}{2}\ \mu v\ \omega^2 y_m^{\ 2} = 10\ \text{W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); P = 20 W.

(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-16(a)) then the amplitude y_m is doubled which means its square $y_m^{\ 2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); P = 40 W.

(d) Eq. 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi)$ = 1.618 times the original amplitude y_m. Squared, this results in an increase in the power by a factor of 2.618. Thus, P = 26 W in this case.

(e) Now the situation depicted in Fig. 16-16(b) applies, so $P = 0$.

38. The nth resonant frequency of string A is

$$f_{n,A} = \frac{v_A}{2l_A} n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string B it is

$$f_{n,B} = \frac{v_B}{2l_B} n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4} f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of B matches the frequency of A's first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of B would match $f_{3,A} = \dfrac{3v_A}{2l_A} = \dfrac{3}{2L}\sqrt{\dfrac{\tau}{\mu}}$,

39. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250 \text{ N}}{(10.0 \text{ m})\,(0.100 \text{ kg})}} = n\ (7.91 \text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91 \text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91 \text{ Hz}) = 23.7$ Hz.

40. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\text{N}}{2.00 \times 10^{-3}\,\text{kg}/1.25\,\text{m}}} = 66.1\,\text{m/s}.$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where L = 125 cm. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1 \text{ m/s}}{2(1.25 \text{ m})} = 26.4 \text{ Hz}.$$

41. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0 \text{ N})\,(8.40 \text{ m})}{0.120 \text{ kg}}} = 82.0 \text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L$ = 2(8.40 m) = 16.8 m.

(c) The frequency is $f = v/\lambda$ = (82.0 m/s)/(16.8 m) = 4.88 Hz.

43. (a) Eq. 16–26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150 \text{ N}}{7.20\times10^{-3} \text{ kg/m}}} = 144.34 \text{ m/s} \approx 1.44\times10^{2} \text{ m/s}.$$

(b) From the Figure, we find the wavelength of the standing wave to be λ = (2/3)(90.0 cm) = 60.0 cm.

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^{2} \text{ m/s}}{0.600 \text{ m}} = 241 \text{ Hz}.$$

44. Use Eq. 16–66 (for the resonant frequencies) and Eq. 16–26 $(v = \sqrt{\tau/\mu})$ to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\,\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3.$$

(b) And we get the new wavelength

$$\lambda_3' = \frac{v'}{f_3'} = \frac{2L}{3} = \lambda_3.$$

45. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\text{ Hz}}{420\text{ Hz} - 315\text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\text{ Hz})/3 = 105\text{ Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then

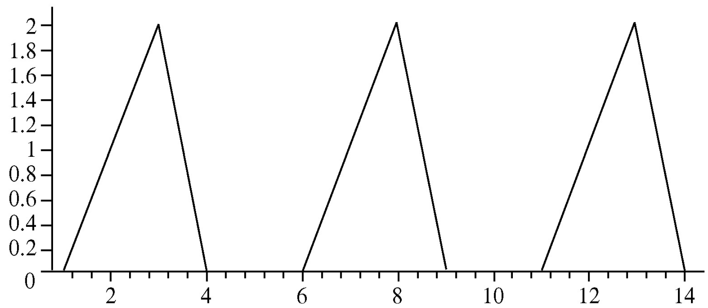
$$v = \lambda f = 2Lf = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

46. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus, $f_1 = (390\text{ Hz} - 325\text{ Hz}) = 65\text{ Hz}$. This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

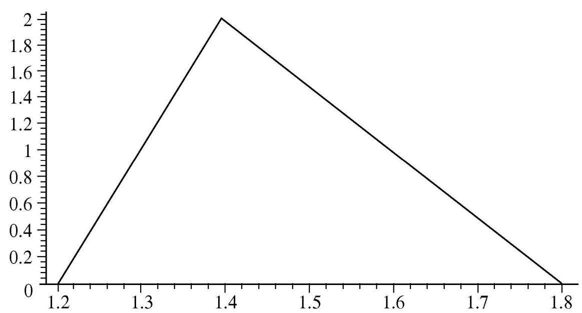
59. (a) Recalling the discussion in §16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where x is in centimeters and t is in seconds) must be 5.0 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16–45 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0\text{ s}$. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.



(d) The leading edge of the pulse reaches x = 10 cm at t = (10 – 4.0)/5 = 1.2 s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement h = 2 cm at t = (10 – 3.0)/5 = 1.4 s. Finally, the trailing edge of the pulse departs from x = 10 cm at t = (10 – 1.0)/5 = 1.8 s. Thus, we find for $h(t)$ at x = 10 cm (with the horizontal axis, t, in seconds):



67. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

68. Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^8 \text{ N/m}^2}{7800\,\text{kg/m}^3}} = 3.00\times10^2 \text{ m/s}$$

which is indeed independent of the diameter of the wire.

83. To oscillate in four loops means n = 4 in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, λ = 2(0.90 m)/4 = 0.45 m. Therefore, the speed of the wave is $v = f\lambda$ = 27 m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044\text{ kg})/(0.90\text{ m}) = 0.049\text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2\,\mu = (27)^2(0.049) = 36\text{ N}.$$

89. (a) For visible light

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^8\text{ m/s}}{700\times10^{-9}\text{m}} = 4.3\times10^{14}\text{ Hz}$$

and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^8\text{ m/s}}{400\times10^{-9}\text{m}} = 7.5\times10^{14}\text{ Hz.}$$

(b) For radio waves

$$\lambda_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^8\text{ m/s}}{300\times10^{6}\text{Hz}} = 1.0\text{m}$$

and

$$\lambda_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^8\text{ m/s}}{1.5\times10^{6}\text{Hz}} = 2.0\times10^{2}\text{m.}$$

(c) For X rays

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^8\text{ m/s}}{5.0\times10^{-9}\text{m}} = 6.0\times10^{16}\text{Hz}$$

and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^8\text{ m/s}}{1.0\times10^{-11}\text{m}} = 3.0\times10^{19}\text{ Hz.}$$

94. We refer to the points where the rope is attached as A and B, respectively. When A and B are not displaced horizontal, the rope is in its initial state (neither stretched (under tension) nor slack).

If they are displaced away from each other, the rope is clearly stretched. When A and B are displaced in the same direction, by amounts (in absolute value) $|\xi_A|$ and $|\xi_B|$, then if $|\xi_A| < |\xi_B|$ then the rope is stretched, and if $|\xi_A| > |\xi_B|$ the rope is slack. We must be careful about the case where one is displaced but the other is not, as will be seen below.

(a) The standing wave solution for the shorter cable, appropriate for the initial condition $\xi = 0$ at $t = 0$, and the boundary conditions $\xi = 0$ at $x = 0$ and $x = L$ (the x axis runs vertically here), is $\xi_A = \xi_m \sin(k_A x) \sin(\omega_A t)$. The angular frequency is $\omega_A = 2\pi/T_A$, and the wave number is $k_A = 2\pi/\lambda_A$ where $\lambda_A = 2L$ (it begins oscillating in its fundamental mode) where the point of attachment is $x = L/2$. The displacement of what we are calling point A at time $t = \eta T_A$ (where η is a pure number) is

$$\xi_A = \xi_m \sin\left(\frac{2\pi}{2L}\frac{L}{2}\right)\sin\left(\frac{2\pi}{T_A}\eta T_A\right) = \xi_m \sin(2\pi\eta).$$

The fundamental mode for the longer cable has wavelength $\lambda_B = 2\lambda_A = 2(2L) = 4L$, which implies (by $v = f\lambda$ and the fact that both cables support the same wave speed v) that $f_B = \frac{1}{2} f_A$ or $\omega_B = \frac{1}{2}\omega_A$. Thus, the displacement for point B is

$$\xi_B = \xi_m \sin\left(\frac{2\pi}{4L}\frac{L}{2}\right)\sin\left(\frac{1}{2}\left(\frac{2\pi}{T_A}\right)\eta T_A\right) = \frac{\xi_m}{\sqrt{2}}\sin(\pi\eta).$$

Running through the possibilities $\left(\eta = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \text{and } 2\right)$ we find the rope is under tension in the following cases. The first case is one we must be very careful about in our reasoning, since A is not displaced but B is displaced in the positive direction; we interpret that as the direction *away from A* (rightwards in the figure) — thus making the rope stretch.

$$\begin{aligned}
\eta &= \frac{1}{2} \quad & \xi_A &= 0 \quad & \xi_B &= \frac{\xi_m}{\sqrt{2}} > 0 \\
\eta &= \frac{3}{4} \quad & \xi_A &= -\xi_m < 0 \quad & \xi_B &= \frac{\xi_m}{2} > 0 \\
\eta &= \frac{7}{4} \quad & \xi_A &= -\xi_m < 0 \quad & \xi_B &= -\frac{\xi_m}{2} < 0
\end{aligned}$$

where in the last case they are both displaced leftward but A more so than B so that the rope is indeed stretched.

(b) The values of η (where we have defined $\eta = t/T_A$) which reproduce the initial state are

$$\begin{aligned}
\eta &= 1 \quad & \xi_A &= 0 \quad & \xi_B &= 0 \quad \text{and} \\
\eta &= 2 \quad & \xi_B &= 0 \quad & \xi_B &= 0.
\end{aligned}$$

(c) The values of η for which the rope is slack are given below. In the first case, both displacements are to the right, but point A is farther to the right than B. In the second case, they are displaced towards each other.

$$\eta=\frac{1}{4} \qquad \xi_A = x_m > 0 \quad \xi_B = \frac{\xi_m}{\sqrt{2}} > 0$$
$$\eta=\frac{5}{4} \qquad \xi_A = \xi_m > 0 \quad \xi_B = -\frac{\xi_m}{2} < 0$$
$$\eta=\frac{3}{2} \qquad \xi_A = 0 \qquad\quad \xi_B = -\frac{\xi_m}{\sqrt{2}} < 0$$

where in the third case B is displaced leftward toward the undisplaced point A.

(d) The first design works effectively to damp fundamental modes of vibration in the two cables (especially in the shorter one which would have an anti-node at that point), whereas the second one only damps the fundamental mode in the longer cable.