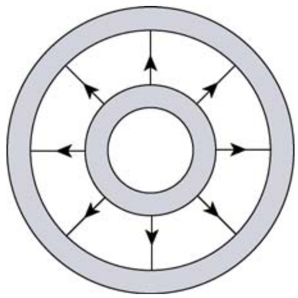
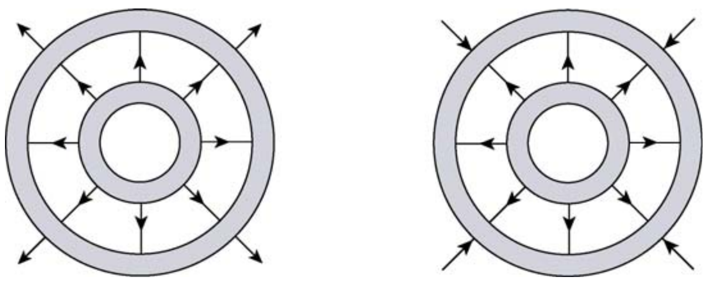
Halliday/Resnick/Walker 7e

Chapter 22 – Electric Fields

1. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge which resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



5. Since the magnitude of the electric field produced by a point charge q is given by $E = |q| / 4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2 (2.0\,\text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times10^{-11}\text{C}.$$

6. With x_1 = 6.00 cm and x_2 = 21.00 cm, the point midway between the two charges is located at x = 13.5 cm. The values of the charge are $q_1 = -q_2 = -2.00 \times 10^{-7}$ C, and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0 (x-x_1)^2}\,\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0 (x-x_1)^2}\,\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5 \text{ N/C})\hat{\text{i}}$$

8. (a) The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 22-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}} \; .$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > L$ region:

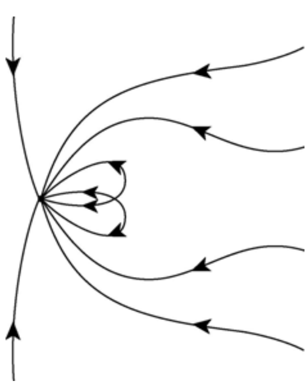
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



9. At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\, q_1$ located at $x_2 = 70$ cm has a greater magnitude than $q_1 = 2.1$

$\times 10^{-8}$ C located at x_1 = 20 cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{\left(x-x_1\right)^2}\right).$$

If the field is to vanish, then

$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{\left(x-x_1\right)^2} \quad \Rightarrow \quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{\left(x-x_1\right)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

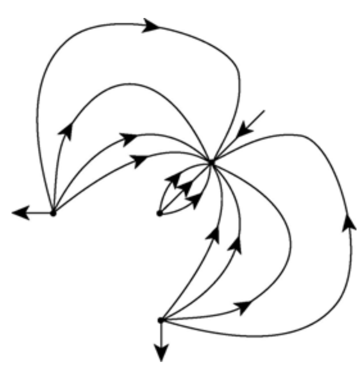
$$\frac{x-70}{x-20} = \pm 2.0\,.$$

Choosing –2.0 for consistency, the value of x is found to be x = –30 cm.

10. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{\left(2d\right)^2} - \frac{q_3}{d^2}\right)\hat{\text{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\text{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown below:



11. The *x* component of the electric field at the center of the square is given by

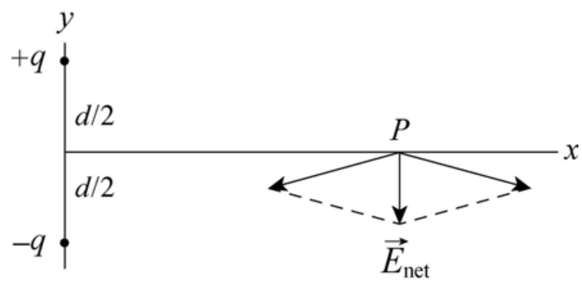
$$
\begin{aligned}
E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= 0.
\end{aligned}
$$

Similarly, the *y* component of the electric field is

$$
\begin{aligned}
E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\ \text{C})}{(0.050\ \text{m})^2/2}\frac{1}{\sqrt{2}} = 1.02\times10^5\ \text{N/C}.
\end{aligned}
$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\ \text{N/C})\hat{\text{j}}$.

19. Consider the figure below.



(a) The magnitude of the net electric field at point *P* is

$$
\left|\vec{E}_{\text{net}}\right| = 2E_1\sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2+r^2}\right]\frac{d/2}{\sqrt{(d/2)^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2+r^2\right]^{3/2}}
$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$
|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.
$$

(b) From the figure, it is clear that the net electric field at point *P* points in the $-\hat{\text{j}}$ direction, or –90° from the +*x* axis.

34. (a) Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \Rightarrow \left|\vec{E}\right| = \frac{mg}{2e}.$$

Using the mass given in the problem, we obtain $\left|\vec{E}\right| = 2.03\times10^{-7}$ N/C.

(b) Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

35. The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ N/C}\right)}{9.11\times10^{-31}\text{ kg}} = 3.51\times10^{15}\text{ m/s}^2 .$$

38. (a) $F_e = Ee = (3.0 \times 10^6\text{ N/C})(1.6 \times 10^{-19}\text{ C}) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = 4.8 \times 10^{-13}$ N.

39. (a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus,

$$E = \frac{F}{q} = \frac{3.0\times10^{-6}\text{ N}}{2.0\times10^{-9}\text{ C}} = 1.5\times10^{3}\text{ N/C}.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_{el} = eE = \left(1.60\times10^{-19}\text{ C}\right)\left(1.5\times10^{3}\text{ N/C}\right) = 2.4\times10^{-16}\text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = \left(1.67\times10^{-27}\text{ kg}\right)\left(9.8\text{ m/s}^2\right) = 1.6\times10^{-26}\text{N}.$$

The force is downward.

(e) The ratio of the forces is

$$\frac{F_{el}}{F_g} = \frac{2.4\times10^{-16}\text{ N}}{1.64\times10^{-26}\text{ N}} = 1.5\times10^{10}.$$

40. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-(9.11\times10^{-31}\text{ kg})(5.00\times10^6\text{ m/s})^2}{-2(1.60\times10^{-19}\text{ C})(1.00\times10^3\text{ N/C})} = 7.12\times10^{-2}\text{ m}.$$

(b) Eq. 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2(7.12\times10^{-2}\text{ m})}{5.00\times10^6\text{ m/s}} = 2.85\times10^{-8}\text{ s}.$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\frac{\Delta K}{K_i} = \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}$$
$$= \frac{-2(1.60\times10^{-19}\text{C})(1.00\times10^3\text{ N/C})(8.00\times10^{-3}\text{m})}{(9.11\times10^{-31}\text{kg})(5.00\times10^6\text{ m/s})^2} = -0.112.$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

41. (a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{(1.60\times10^{-19}\text{ C})(2.00\times10^4\text{ N/C})}{1.67\times10^{-27}\text{ kg}} = 1.92\times10^{12}\text{ m/s}^2\ .$$

(b) We assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2(1.92\times10^{12}\text{ m/s}^2)(0.0100\text{ m})} = 1.96\times10^5\text{ m/s}.$$

43. (a) We use $\Delta x = v_{\text{avg}}t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2(2.0\times10^{-2}\text{ m})}{1.5\times10^{-8}\text{ s}} = 2.7\times10^6\text{ m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta x m}{et^2} = \frac{2(2.0\times10^{-2}\,\text{m})(9.11\times10^{-31}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(1.5\times10^{-8}\,\text{s})^2} = 1.0\times10^3\ \text{N/C}.$$

45. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$\begin{aligned} x &= \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p)+(eE/m_e)}L = \frac{m_e}{m_e+m_p}L \\ &= \frac{9.11\times10^{-31}\,\text{kg}}{9.11\times10^{-31}\,\text{kg}+1.67\times10^{-27}\,\text{kg}}(0.050\,\text{m}) \\ &= 2.7\times10^{-5}\,\text{m}. \end{aligned}$$

59. The distance from Q to P is $5a$, and the distance from q to P is $3a$. Therefore, the magnitudes of the individual electric fields are, using Eq. 22-3 (writing $1/4\pi\varepsilon_0 = k$),

$$\left|\vec{E}_Q\right| = \frac{k|Q|}{25\,a^2}, \quad \left|\vec{E}_q\right| = \frac{k|q|}{9\,a^2}.$$

We note that $\vec{E}_q$ is along the y axis (directed towards $\pm y$ in accordance with the sign of q), and $\vec{E}_Q$ has x and y components, with $\vec{E}_{Q\,x} = \pm\frac{4}{5}\left|\vec{E}_Q\right|$ and $\vec{E}_{Q\,y} = \pm\frac{3}{5}\left|\vec{E}_Q\right|$ (signs corresponding to the sign of Q). Consequently, we can write the addition of components in a simple way (basically, by dropping the absolute values):

$$\begin{aligned} \vec{E}_{\text{net}\,x} &= \frac{4\,k\,Q}{125\,a^2} \\ \vec{E}_{\text{net}\,y} &= \frac{3\,k\,Q}{125\,a^2} + \frac{k\,q}{9\,a^2} \end{aligned}$$

(a) Equating $\vec{E}_{\text{net}\,x}$ and $\vec{E}_{\text{net}\,y}$, it is straightforward to solve for the relation between Q and q. We obtain $Q/q = 125/9 \approx 14$.

(b) We set $\vec{E}_{\text{net}\,y} = 0$ and find the necessary relation between Q and q. We obtain $Q/q = -125/27 \approx -4.6$.

70. The two closest charges produce fields at the midpoint which cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r = \sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E = \frac{Q}{4\pi\varepsilon_o r^2} = \frac{Q}{3\pi\varepsilon_o d^2} .$$

75. (a) Using the density of water (ρ = 1000 kg/m^3), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\ \text{m}\right)^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87\times10^{-15}\ \text{N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)} = 120.$$

79. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\,\text{kg}}\right)\left(1.40\times10^{6}\ \frac{\text{N}}{\text{C}}\right) = 2.46\times10^{17}\ \text{m/s}^2 .$$

(b) With $v = \frac{c}{10} = 3.00\times10^{7}\ \text{m/s}$, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00\times10^{7}}{2.46\times10^{17}} = 1.22\times10^{-10}\ \text{s}.$$

(c) Eq. 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00\times10^{7}\right)^2}{2\left(2.46\times10^{17}\right)} = 1.83\times10^{-3}\ \text{m}.$$

82. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock ($-q$) and seven o'clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock ($-6q$) and twelve o'clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock … twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points towards the nine-thirty position.