Halliday/Resnick/Walker 7e
Chapter 23 – Gauss' Law

21. The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 23-12. Thus,

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.5\times10^{4}\ \mathrm{N/C}\right)\left(2.0\ \mathrm{m}\right) = 5.0\times10^{-6}\ \mathrm{C/m}.$$

22. We combine Newton's second law ($F = ma$) with the definition of electric field ($F = qE$) and with Eq. 23-12 (for the field due to a line of charge). In terms of magnitudes, we have (if r = 0.080 m and $\lambda = 6.0 \times 10^{-6}$ C/m)

$$ma = eE = \frac{e\lambda}{2\pi\varepsilon_o r} \quad \Rightarrow \quad a = \frac{e\lambda}{2\pi\varepsilon_o r\, m} = 2.1\times10^{17}\ \mathrm{m/s^2}\ .$$

24. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{\lambda_1}{2\pi\varepsilon_o(x + L/2)} + \frac{\lambda_2}{2\pi\varepsilon_o(x - L/2)}\ .$$

Setting this equal to zero and solving for x we find

$$x = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\,\frac{L}{2}$$

which, for the values given in the problem, yields x = 8.0 cm.

31. (a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus,

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\ \mathrm{C}}{2(0.080\ \mathrm{m})^2} = 4.69\times10^{-4}\ \mathrm{C/m^2}.$$

The magnitude of the field is

$$E = \frac{4.69\times10^{-4}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^{7}\ \text{N/C}.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where *r* is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\ \text{C}\right)}{\left(30\ \text{m}\right)^2} = 60\ \text{N/C}.$$

32. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77\times10^{-22})/(8.85\times10^{-12}) = 2.00\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\ \text{N/C})\hat{\text{j}}$.

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\ \text{N/C})\hat{\text{j}}$.

35. We use Eq. 23-13.

(a) To the left of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the right plate)} + (\sigma/2\varepsilon_0)\hat{\text{i}}\ \text{(from the left one)} = 0.$$

(b) To the right of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)\hat{\text{i}}\ \text{(from the right plate)} + \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\text{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)\left(-\hat{\text{i}}\right) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\text{i}}) = -\left(\frac{7.00\times10^{-22}\ \text{C/m}^2}{8.85\times10^{-12}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}}\right)\hat{\text{i}} = \left(-7.91\times10^{-11}\ \text{N/C}\right)\hat{\text{i}}.$$

36. The field due to the sheet is $E = \frac{\sigma}{2\varepsilon_o}$. The force (in magnitude) on the electron (due to that field) is $F = eE$, and assuming it's the *only* force then the acceleration is

$$a = \frac{e\sigma}{2\varepsilon_0 m} = \text{slope of the graph } (= 2.0 \times 10^5 \text{ m/s divided by } 7.0 \times 10^{-12} \text{ s}) \; .$$

Thus we obtain $\sigma = 2.9 \times 10^{-6}$ C/m^2.

37. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2} m v_0^2$ is the initial kinetic energy K_0, so

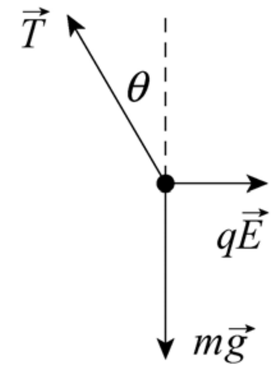
$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.60\times10^{-17} \text{ J}\right)}{\left(1.60\times10^{-19} \text{ C}\right)\left(2.0\times10^{-6} \text{ C/m}^2\right)} = 4.4\times10^{-4} \text{ m}.$$

38. We use the result of part (c) of problem 35 to obtain the surface charge density.

$$E = \sigma/\varepsilon_0 \Rightarrow \sigma = \varepsilon_0 E = \left(8.85\times10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(55 \text{ N/C}) = 4.9\times10^{-10} \text{ C/m}^2.$$

Since the area of the plates is $A = 1.0 \text{ m}^2$, the magnitude of the charge on the plate is $Q = \sigma A = 4.9\times10^{-10}$ C.

39. The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T \sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N.m}^2\right)\left(1.0\times10^{-6}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\tan 30°}{2.0\times10^{-8}\ \text{C}}$$
$$= 5.0\times10^{-9}\ \text{C/m}^2.$$

57. (a) We use $m_e g = eE = e\sigma/\varepsilon_0$ to obtain the surface charge density.

$$\sigma = \frac{m_e g\varepsilon_0}{e} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(9.8\,\text{m/s}\right)\left(8.85\times10^{-12}\ \frac{C^2}{\text{N.m}^2}\right)}{1.60\times10^{-19}\ \text{C}} = 4.9\times10^{-22}\ \text{C/m}^2.$$

(b) Downward (since the electric force exerted on the electron must be upward).

70. Since the fields involved are uniform, the precise location of P is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\text{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}}{2\times8.85\times10^{-12}} = 5.65\times10^4\ \text{N/C}.$$

In unit-vector notation, we have $\vec{E} = (5.65\times10^4\ \text{N/C})\hat{\text{j}}$.