Halliday/Resnick/Walker 7e
Chapter 26 – Current and Resistance

3. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where r is the radius of the sphere. This means

$$\Delta q = 4\pi\varepsilon_0 r\,\Delta V .$$

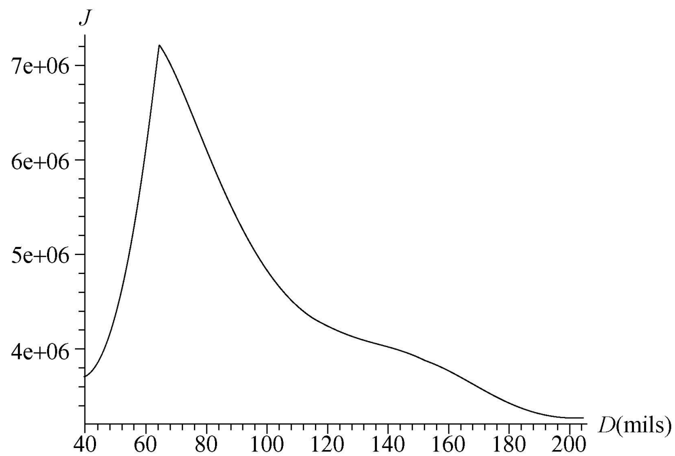
Now, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{\text{in}} - i_{\text{out}}} \\ &= \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times 10^9\text{ F/m})(1.0000020\text{ A} - 1.0000000\text{ A})} = 5.6\times 10^{-3}\text{ s.}\end{aligned}$$

4. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with D = 64 mil = 0.0016 m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6$ A/m^2. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



5. (a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\ \text{m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C},$$

and the drift speed is 1.0×10^5 m/s. Thus,

$$J = \left(2 \times 10^{14} / \text{m}\right)\left(3.2 \times 10^{-19}\ \text{C}\right)\left(1.0 \times 10^5\ \text{m/s}\right) = 6.4\ \text{A/m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is $J = i / A = i / \pi r^2$, so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50\ \text{A}}{\pi\left(440 \times 10^4\ \text{A/m}^2\right)}} = 1.9 \times 10^{-4}\ \text{m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4}\ \text{m}) = 3.8 \times 10^{-4}$ m.

8. (a) Since $1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6}\ \text{m}^3}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(470 \times 10^3\ \text{m/s}\right) = 6.54 \times 10^{-7}\ \text{A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi\left(6.37\times10^6\text{ m}\right)^2\left(6.54\times10^{-7}\text{ A/m}^2\right) = 8.34\times10^7\text{ A}.$$

13. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\text{ m})(4.0\text{ A})}{(2.0\text{ V})\left(1.0\times10^{-6}\text{ m}^2\right)} = 2.0\times10^6/\Omega\cdot\text{m}.$$

15. The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area. In this case,

$$A = \pi r^2 = \pi\left(0.50\times10^{-3}\text{ m}\right)^2 = 7.85\times10^{-7}\text{ m}^2.$$

Thus,

$$\rho = \frac{RA}{L} = \frac{\left(50\times10^{-3}\Omega\right)\left(7.85\times10^{-7}\text{ m}^2\right)}{2.0\text{m}} = 2.0\times10^{-8}\Omega\cdot\text{m}.$$

17. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50\times10^{-3}\text{ A})(2000\ \Omega) = 100$ V.

18. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R.$$

19. The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12\text{ m}) = 188.5\text{ m}.$$

If r_w is the radius of the wire itself, then its cross-sectional area is $A = \pi r^2 w = \pi(0.65\times10^{-3}\text{ m})^2 = 1.33\times10^{-6}\text{ m}^2$. According to Table 26-1, the resistivity of copper is $1.69\times10^{-8}\Omega\cdot\text{m}$. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(188.5\text{ m}\right)}{1.33\times10^{-6}\text{ m}^2} = 2.4\,\Omega.$$

21. Since the mass density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0,$$

where R_0 is the original resistance. Thus, $R = 9(6.0\ \Omega) = 54\ \Omega$.

25. The resistance at operating temperature T is $R = V/i = 2.9\text{ V}/0.30\text{ A} = 9.67\ \Omega$. Thus, from $R - R_0 = R_0\alpha(T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \left(\frac{1}{4.5\times10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) = 1.9\times10^3\ °\text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

28. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) $J = \sigma E = (2.70 \times 10^{-14}\ /\ \Omega\cdot\text{m})\ (120\text{ V/m}) = 3.24 \times 10^{-12}\text{ A/m}^2$.

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{\left(2.70\times10^{-14}/\Omega\cdot\text{m}\right)\left(120\text{ V/m}\right)}{\left[\left(620+550\right)/\text{cm}^3\right]\left(1.60\times10^{-19}\text{ C}\right)} = 1.73\text{ cm/s}.$$

35. (a) Electrical energy is converted to heat at a rate given by

$$P = \frac{V^2}{R},$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\text{ V})^2}{14\ \Omega} = 1.0\times10^3\text{ W} = 1.0\text{ kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = \text{US\$}0.25$.

36. Since $P = iV$, $q = it = Pt/V = (7.0\text{ W})\,(5.0\text{ h})\,(3600\text{ s/h})/9.0\text{ V} = 1.4 \times 10^4$ C.

37. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\text{ V}}{3.00\text{ V}}\right)^2 (0.540\text{ W}) = 0.135\text{ W}.$$

42. The slopes of the lines yield $P_1 = 8$ mW and $P_2 = 4$ mW. Their sum (by energy conservation) must be equal to that supplied by the battery: $P_{\text{batt}} = (\,8 + 4\,)$ mW = 12 mW.

49. (a) We are told that $r_B = \frac{1}{2} r_A$ and $L_B = 2L_A$. Thus, using Eq. 26-16,

$$R_B = \rho \frac{L_B}{\pi r_B^2} = \rho \frac{2LA}{\frac{1}{4}\pi r_A^2} = 8R_A = 64\,\Omega.$$

(b) The current densities are assumed uniform.

$$\frac{J_A}{J_B} = \frac{i/\pi r_A^2}{i/\pi r_B^2} = \frac{i/\pi r_A^2}{4i/\pi r_A^2} = 0.25.$$

54. Since 100 cm = 1 m, then $10^4\text{ cm}^2 = 1\text{ m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{\left(3.00 \times 10^{-7}\,\Omega\cdot\text{m}\right)\left(10.0 \times 10^3\text{ m}\right)}{56.0 \times 10^{-4}\text{ m}^2} = 0.536\,\Omega.$$

58. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\,\Delta V$. The percentage change is roughly

$$\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%.$$

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

61. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0\,\text{A})(120\,\text{V}) = 1.20\,\text{kW}.$$

Dividing this into the 180 kJ necessary to cook the three hot-dogs leads to the result $t = 150$ s.

63. We use $P = i^2 R = i^2 \rho L/A$, or $L/A = P/i^2\rho$.

(a) The new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}}.$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875}L_{\text{old}} = 1.37L_{\text{old}} \quad \Rightarrow \quad \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37\,.$$

(b) Similarly, we note that $(LA)_{\text{new}} = (LA)_{\text{old}}$, and

$$A_{\text{new}} = \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}} \quad \Rightarrow \quad \frac{A_{\text{new}}}{A_{\text{old}}} = 0.730\,.$$

68. We use Eq. 26-28:

$$R = \frac{V^2}{P} = \frac{200^2}{3000} = 13.3\ \Omega \quad .$$

72. Combining Eq. 26-28 with Eq. 26-16 demonstrates that the power is inversely proportional to the length (when the voltage is held constant, as in this case). Thus, a new length equal to 7/8 of its original value leads to

$$P = \frac{8}{7}\ (2.0\text{ kW}) = 2.4\text{ kW} \quad .$$