PHY 106 Final Exam May 3, 2006

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1. A cube of Plexiglas has a critical angle for total internal reflection of 35° when in air. (**8 pts**)
   a) What is the index of refraction of the Plexiglas?
   b) What is the critical angle in Plexiglas if it is submerged in water (n = 1.3)?

a) The critical angle for total internal reflection is determined by $n_1 \sin\theta_1 = n_2 \sin 90^\circ$. In this problem, n_2 = 1, and therefore $n_1 = \dfrac{1}{\sin 35^\circ} = 1.743$

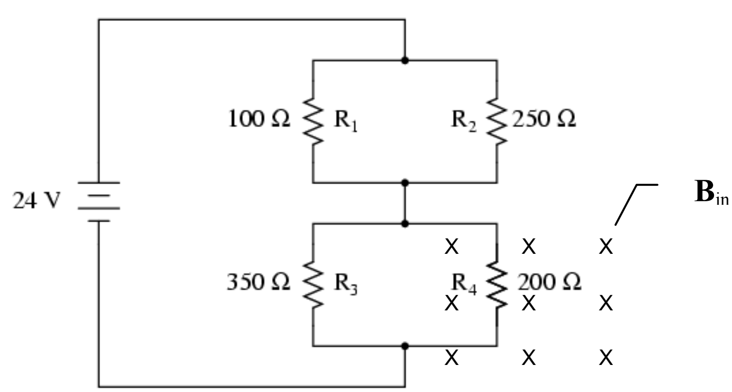
b) Using the same formula from (a), in this problem n_2 = 1.3. The critical angle is then $\theta_1 = \sin^{-1}\dfrac{n_2}{n_1} = 48.21^\circ$.

2. A sinusoidal wave travels on a wire in the negative y-direction with wavelength 50cm, speed 4.0 m/s, and amplitude 5.0 cm. (**8 pts**)
   a) Write down the wave equation (in MKS units)
   b) Determine the maximum vertical acceleration of any point on the wire

a) Using $\mathrm{v} = \dfrac{\omega}{\mathrm{k}}$ and $k = \dfrac{2\pi}{\lambda}$ and the given values for v and wavelength,
$y = 0.05\sin(4\pi\, x + 16\pi\, t)$ m

b) Find acceleration by taking the second derivative of y with respect to t. The maximum acceleration is the amplitude of the resulting sine function: $a_{\max} = 0.05(16\pi)^2 = 126.33$ m/s^2

3. In the following circuit, R_4 sits in a 50 mT uniform B-field. What are the magnitude and direction of the net magnetic force on the circuit? (R_4 is 10 cm long.) (**12 pts**)



To find the magnetic force, the current through R_4 must be calculated. The two parallel circuit pairs can be replaced with individual resistors: $R_{1P2} = 71.43\ \Omega$ and $R_{3P4} = 127.27\ \Omega$. These equivalent resistors are now in series, i.e. a voltage-divider configuration. The voltage across the R_3-R_4 combination, which is the voltage across R_4 is therefore $\Delta V_4 = \dfrac{127.27}{127.27 + 71.43} \cdot 24 = 15.37\ \text{V}$.

The current through R_4 is $i_4 = \dfrac{15.37}{200} = 0.0769\ \text{A}$. The positive current moves down through R_4 and is perpendicular to B. The magnetic force is therefore $i_4 LB = 3.84 \times 10^{-4}\ \text{N}$, directed to the right.

4. After charging is completed in the following circuit, the charge on C_2 is 450 μC. What is the capacitance of C_2? What is the total stored energy in the capacitors? (**8 pts**)

Capacitors in series carry the same charge. This fact allows the voltage across the 25 μF capacitor to be calculated: $\Delta V = \dfrac{Q}{C} = \dfrac{450}{25} = 18\ \text{V}$. The voltage across C_2 is then 60-18 = 42 V and therefore $C_2 = \dfrac{450}{42} = 10.714\ \mu\text{F}$.

Using $U = \frac{1}{2} C \Delta V^2$, the total stored energy in C_1 is $\frac{1}{2} \times 25 \times (18)^2 = 4.05\ \text{mJ}$ and the total stored energy in C_2 is $\frac{1}{2} \times 10.714 \times (42)^2 = 9.45\ \text{mJ}$, for a net total of 13.5 mJ.

5. An ultrasonic transducer for ultrasound imaging is a thin, 0.1 g disk that is driven back and forth in SHM at 1.0 MHz by an electromagnetic coil. (**8 pts**)

   a) The maximum restoring force on the disk before breaking is 40,000 N. What is the maximum oscillation amplitude that won't rupture the disk?

   b) What is the disk's maximum speed at this amplitude?

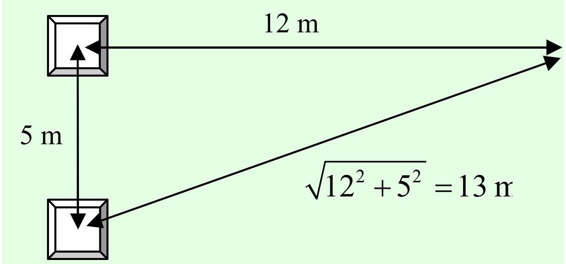
a) Treating the transducer as a mass on a spring, the spring constant is found using $\omega = \sqrt{\dfrac{k}{m}}$, yielding $k = \omega^2 m = 4\pi^2 \times 10^8\ \text{N/m}$. With SHM, there is a linear restoring force. Therefore, the maximum amplitude is found using Hooke's Law: $x_{\max} = \dfrac{F_{\max}}{k} = 1.013 \times 10^{-5}\ \text{m}$

b) The maximum speed of a mass undergoing SHM is $v_{\max} = x_{\max}\omega = 63.7\ \text{m/s}$

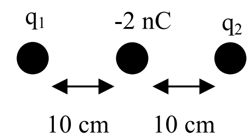
6. Two loudspeakers 5.0 m apart are playing the same frequency. If you stand 12.0 m in front of the speakers, centered between them, you hear maximum intensity. As you walk parallel to the speaker plane (staying 12.0 m in front of them), you first hear a minimum intensity when you are directly in front of one of the speakers. What is the frequency of the sound? (**6 pts**)

The first minimum in intensity occurs when the path length difference is $\lambda/2$. From the diagram, the path length difference is 13-12 = 1.0 m and therefore λ =2.0 m. Taking the velocity of sound in air to be 343 m/s, the frequency is

$$f = \frac{v}{\lambda} = \frac{343}{2} = 171.5 \text{ Hz}$$



7. In the following, charge q_2 is in static equilibrium. (**10 pts**)
   a) What is q_1?
   b) What is the net E-field on the middle charge?



a) The force on q_2 due to q_1 and the middle charge must be equal in magnitude, but opposite in sign. Setting d = 0.1 m and q_{mid} = 2.0 nC,

$$k\frac{q_1 q_2}{(2d)^2} = k\frac{q_{mid} q_2}{d^2} \quad \text{or} \quad q_1 = 4q_{mid} = 8.0 \text{ nC}$$

b) With the positive x-axis to the right, the net E-field on the middle charge is the vector sum of the E-fields due to q_1 and q_2. (Also note that the E-field from q_2 must include a negative sign because q_2 is on the RHS). The total field is therefore

$$\vec{\mathrm{E}}_{Net} = k\frac{q_1}{d^2}\hat{i} + (-)k\frac{q_2}{d^2}\hat{i} = k\frac{(q_1 - q_2)}{d^2}\hat{i}$$

DO TWO OF THE FOLLOWING THREE!!!! (Each worth 10 points)

8. The electron gun in a TV tube uses a uniform E-field to accelerate electrons from rest to 5.0×10^7 m/s in a distance of 1.2 cm. What is the strength of the E-field?

The gain in kinetic energy is due to a loss in electrostatic potential energy:

$$\frac{1}{2}mv^2 = eEd \text{ , or } E = \frac{mv^2}{2ed} = 5.92 \times 10^5 \text{ N/C}$$

9. A bat locates insects by emitting ultrasonic chirps, and then listening for echoes. If the bat chirps are 25 kHz, how fast would the bat have to fly, and in what direction, for a human to barely be able to hear the chirp? (Human hearing cuts off at approx. 20 kHz).

The bat must be flying away from the observer if there is a decrease in frequency. Using the Doppler formula for moving source, and v = 343 m/s the speed of sound in air,

$20 = \left(\frac{343}{343+v_s}\right)25$. Solving for v_s yields $v_s = 85.75$ m/s

10. A heavy piece of hanging sculpture is suspended by a 90-cm-long, 5.0 g steel wire. When the wind blows hard, the wire hums at its fundamental frequency of 80 Hz. What is the mass of the sculpture?

Using the fact that the wavelength is 2 × *Length* for the fundamental, the velocity of the wave in the string is $v = f\lambda = 144$ m/s. On a string, the speed is given by $v = \sqrt{\frac{Tension}{\mu}} = \sqrt{\frac{mg}{\mu}}$.
Therefore, $\sqrt{\frac{mg}{\mu}} = 144$, or $m = \frac{144^2\mu}{g}$. In this problem $\mu = \frac{5\times10^{-3}}{0.9} = 5.56\times10^{-3}$ kg/m, and the mass of the sculpture is $m = 11.76$ kg.

$$f' = \left(\frac{v}{v \mp v_s}\right)f \qquad f' = \left(\frac{v \pm v_o}{v}\right)f$$